MATH 250 HANDOUT 13 - 'FIXING' THE HYPOTHESES OF STATEMENTS

(1) Let $f\colon A \to B$ be a function and let $X, Y \subset A$ and let $W, V \subset B$. Each of the following statements are false as stated. Which become true if we assume that f is injective or surjective? In each case (f is injective, or f is surjective), prove your assertion or give a counterexample.
   (a) $X \subset Y \Leftarrow f(X) \subset f(Y)$.
   (b) $f(X \cap Y) \supset f(X) \cap f(Y)$.
   (c) $f(X) - f(Y) \supset f(X - Y)$.
   (d) $X \supset f^{-1}(f(X))$.
   (e) $W \subset f(f^{-1}(W))$.
   (f) $V \subset W \Leftarrow f^{-1}(V) \subset f^{-1}(W)$.