MATH 250 HANDOUT 14 - COMPOSITIONS AND INJECTIVITY/SURJECTIVITY

(1) Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1+x^2}$ and let $g\colon \mathbb{R} \to \mathbb{R}$ be the function $g(x) = e^x$.

(a) What is $g \circ f(0)$?
(b) What is $f \circ g(0)$?
(c) Give a formula for $f \circ g$ and $g \circ f$.

(2) Let $f\colon \mathbb{R} \to \mathbb{Z}$ be the function $f(x) = \lfloor x \rfloor$ (i.e., round x down to the nearest integer) and let $g\colon \mathbb{Z} \to \mathbb{Z}$ be the function $g(n) =$ 'the number of distinct prime factors of n'. (So $g(0) = g(1) = 0$, $g(4) = 1$, $g(6) = 2$))

(a) What is $g \circ f(\pi)$?
(b) What is $g \circ f(91.1023124)$?
(c) Is $g \circ f$ injective? Surjective?

(3) Let $f\colon \mathbb{Z} \to P(\mathbb{Z})$ be the function $f(n) = n$ and let $g\colon P(\mathbb{Z}) \to P(\mathbb{Z})$ be the function $g(S) = S \cap \{1\}$.

(a) What is $g \circ f(0)$?
(b) What is $g \circ f(1)$?
(c) Give a formula for $g \circ f$.

(4) Let $f\colon A \to B$ and $g\colon B \to C$ be functions. Prove or disprove each of the following:

(a) If f and g are injections, then gf is an injection.
(b) If f and g are surjections, then gf is a surjection.
(c) If f and g are bijections, then gf is a bijection.
(d) If gf is an injection, then f and g are injections.
(e) If gf is a surjection, then f and g are surjections.
(f) If gf is a bijection, then f and g are bijections.
(g) If gf is an injection, then f is an injection.
(h) If gf is an injection, then g is an injection.
(i) If gf is a surjection, then f is a surjection.
(j) If gf is a surjection, then g is a surjection.
(k) If gf is a bijection, then f is a bijection.
(l) If gf is a bijection, then g is a bijection.
(m) If gf is an injection and g is invertible, then f is an injection.