

## MATH 250 HANDOUT 15 - BINARY OPERATIONS

(1) Which of these are binary operations? Which are commutative/associative, which have an identity element, and for which does every element have an inverse?

- (a)  $\mathbb{Z} \times \mathbb{Z} \xrightarrow{+} \mathbb{Z}$ .
- (b)  $\mathbb{R} \times \mathbb{R} \xrightarrow{-} \mathbb{R}$ .
- (c)  $\mathbb{R} \times \mathbb{R} \xrightarrow{/} \mathbb{R}$ .
- (d)  $\mathbb{R}^* \times \mathbb{R}^* \xrightarrow{/} \mathbb{R}^*$  (where  $\mathbb{R}^* = \mathbb{R} - \{0\}$ .)
- (e)  $Fun(B, B) \times Fun(B, B) \xrightarrow{\circ} Fun(B, B)$ .
- (f)  $\mathbb{Z} \times \mathbb{Z} \xrightarrow{*} \mathbb{Z}$  (where  $a * b = a + b + 1$ ).
- (g)  $\mathbb{Z} \times \mathbb{Z} \xrightarrow{*} \mathbb{Z}$  (where  $a * b = 2a + b$ ).
- (h)  $P(A) \times P(A) \xrightarrow{\cap} P(A)$ .
- (i)  $P(A) \times P(A) \xrightarrow{\cup} P(A)$ .
- (j)  $\mathbb{R} \cup \{\infty\} \times \mathbb{R} \cup \{\infty\} \xrightarrow{\oplus} \mathbb{R} \cup \{\infty\}$  (where  $a \oplus b = \max(a, b)$ .)
- (k)  $\{0, 1\} \times \{0, 1\} \xrightarrow{*} \{0, 1\}$  (where  $0 * 0 = 0, 0 * 1 = 1, 1 * 0 = 0, 1 * 1 = 0$ ).

Answers (please circle):

- |     |             |             |          |          |
|-----|-------------|-------------|----------|----------|
| (a) | commutative | associative | identity | inverses |
| (b) | commutative | associative | identity | inverses |
| (c) | commutative | associative | identity | inverses |
| (d) | commutative | associative | identity | inverses |
| (e) | commutative | associative | identity | inverses |
| (f) | commutative | associative | identity | inverses |
| (g) | commutative | associative | identity | inverses |
| (h) | commutative | associative | identity | inverses |
| (i) | commutative | associative | identity | inverses |
| (j) | commutative | associative | identity | inverses |
| (k) | commutative | associative | identity | inverses |

(2) Try to think of 3 more examples of binary operations.