MATH 250 HANDOUT 12 - IMAGES AND PREIMAGES

Let $f\colon A \to B$ be a function and let $X, Y \subset A$. Prove or disprove each of the following:

(1) $X \subset Y \Rightarrow f(X) \subset f(Y)$.
(2) $X \subset Y \Leftarrow f(X) \subset f(Y)$.
(3) $f(X \cup Y) \subset f(X) \cup f(Y)$.
(4) $f(X \cup Y) \supset f(X) \cup f(Y)$.
(5) $f(X \cap Y) \subset f(X) \cap f(Y)$.
(6) $f(X \cap Y) \supset f(X) \cap f(Y)$.
(7) $f(X) - f(Y) \subset f(X - Y)$.
(8) $f(X) - f(Y) \supset f(X - Y)$.

Let $f\colon A \to B$ be a function, let $W \subset A$, and let $X, Y \subset B$. Prove or disprove each of the following:

(9) $f^{-1}(X \cup Y) \subset f^{-1}(X) \cup f^{-1}(Y)$.
(10) $f^{-1}(X \cup Y) \supset f^{-1}(X) \cup f^{-1}(Y)$.
(11) $f^{-1}(X \cap Y) \subset f^{-1}(X) \cap f^{-1}(Y)$.
(12) $f^{-1}(X \cap Y) \supset f^{-1}(X) \cap f^{-1}(Y)$.
(13) $W \subset f^{-1}(f(W))$.
(14) $W \supset f^{-1}(f(W))$.
(15) $X \subset f(f^{-1}(X))$.
(16) $X \supset f(f^{-1}(X))$.