

MATH 250 HANDOUT ?? - BIJECTIONS

- (1) For each of the following, give an example of a bijection. Once you have done this, prove that each of your bijections is in fact a bijection.
- (a) $\mathbb{Z} \rightarrow \mathbb{Z} \times \{1, 2\}$.
 - (b) $2\mathbb{Z} \rightarrow 2\mathbb{Z} \times \{1, 2\}$.
 - (c) $\mathbb{Z} \rightarrow \mathbb{Z} \times \{1, 2, 3\}$.
 - (d) $\mathbb{O} \rightarrow \mathbb{E} \times \{1, 2, 3\}$.
 - (e) $\mathbb{Z} \rightarrow d\mathbb{Z}$, where d is an integer.
 - (f) $\mathbb{Z} \rightarrow \mathbb{Z}_{\geq 0}$.
 - (g) $\mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$.
 - (h) $\mathbb{Z} \rightarrow \mathbb{Q}$ (Hint: Modify the bijection $\mathbb{Z}_{>0} \rightarrow \mathbb{Q}_{>0}$ from class.)
- (2) For each of the following, give an example of an injection or a surjection.
- (a) $\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ (injection).
 - (b) $\mathbb{Z}^d \rightarrow \mathbb{Z}$ (injection).
 - (c) $\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Q}$ (surjection).
- (3) Prove that the relation $A \sim B$ if there exists a bijection from A to B is an equivalence relation.