

MATH 250 HANDOUT 16 - BINARY OPERATIONS

(1) Which of these are binary operations? Which are commutative/associative, which have an identity element, and for which does every element have an inverse?

- (a) $\mathbb{Z} \times \mathbb{Z} \xrightarrow{+} \mathbb{Z}$.
- (b) $\mathbb{R} \times \mathbb{R} \xrightarrow{-} \mathbb{R}$.
- (c) $\mathbb{R} \times \mathbb{R} \xrightarrow{/} \mathbb{R}$.
- (d) $\mathbb{R}^* \times \mathbb{R}^* \xrightarrow{/} \mathbb{R}^*$ (where $\mathbb{R}^* = \mathbb{R} - \{0\}$.)
- (e) $Fun(B, B) \times Fun(B, B) \xrightarrow{\circ} Fun(B, B)$.
- (f) $\mathbb{Z} \times \mathbb{Z} \xrightarrow{+} \mathbb{Z}$ (where $a \star b = a + b + 1$).
- (g) $\mathbb{Z} \times \mathbb{Z} \xrightarrow{+} \mathbb{Z}$ (where $a \star b = 2a + b$).
- (h) $P(A) \times P(A) \xrightarrow{\cap} P(A)$.
- (i) $P(A) \times P(A) \xrightarrow{\cup} P(A)$.
- (j) $\mathbb{R} \cup \{\infty\} \times \mathbb{R} \cup \{\infty\} \xrightarrow{\oplus} \mathbb{R} \cup \{\infty\}$ (where $a \oplus b = \max(a, b)$.)
- (k) $\{0, 1\} \times \{0, 1\} \xrightarrow{\star} \{0, 1\}$ (where $0 \star 0 = 0, 0 \star 1 = 1, 1 \star 0 = 0, 1 \star 1 = 0$).

Answers (please circle):

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|-----|-------------|-------------|----------|----------|
| (a) | commutative | associative | identity | inverses |
| (b) | commutative | associative | identity | inverses |
| (c) | commutative | associative | identity | inverses |
| (d) | commutative | associative | identity | inverses |
| (e) | commutative | associative | identity | inverses |
| (f) | commutative | associative | identity | inverses |
| (g) | commutative | associative | identity | inverses |
| (h) | commutative | associative | identity | inverses |
| (i) | commutative | associative | identity | inverses |
| (j) | commutative | associative | identity | inverses |
| (k) | commutative | associative | identity | inverses |

(2) Try to think of 3 more examples of binary operations.