

## MATH 250 HANDOUT 5 - INDUCTION WARMUP

1. We want to prove, by induction, that, for every positive integer  $n$ ,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

- a) What is the statement “ $P(n)$ ”?

$$P(n) =$$

- b) What is the statement “ $P(1)$ ”? Why is  $P(1)$  true?

$$P(1) =$$

- c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that  $P(n-1) \Rightarrow P(n)$ ).

Assume that

We want to show that

(Inductive step)

2. Let  $a_n$  be a sequence such that  $a_1 = 1$  and  $a_n = na_{n-1}$ . We want to prove, by induction, that, for every positive integer  $n$ ,

$$a_n = n! = n(n-1)(n-2) \cdots 2 \cdot 1.$$

- a) What is the statement “ $P(n)$ ”?

$$P(n) =$$

- b) What is the statement “ $P(1)$ ”? Why is  $P(1)$  true?

$$P(1) =$$

- c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that  $P(n-1) \Rightarrow P(n)$ ).

Assume that

We want to show that

(Inductive step)

3. We want to prove, by induction, that, for every positive integer  $n$ ,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

a) What is the statement “ $P(n)$ ”?

$$P(n) =$$

b) What is the statement “ $P(1)$ ”? Why is  $P(1)$  true?

$$P(1) =$$

c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that  $P(n-1) \Rightarrow P(n)$ ).

Assume that

We want to show that

(Inductive step)

4. Prove, by induction, that  $2^{n+1} \geq n^2$  for every integer  $n$ . (For this problem, you will have to first check  $P(1)$  and  $P(2)$ .)