MATH 250 HANDOUT 3 - PROOF BY CONTRADICTION

(1) Prove that if $x + y > 5$, then $x > 2$ or $y > 3$.
(2) Let $0 < \alpha <1$. Prove that $\sqrt{\alpha} > \alpha$.
(3) Prove that there are no integer solutions to the equation $x^2 = 4y + 2$
(4) Prove that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.
(5) Prove that there is no smallest positive real number.
(6) Let b_1, b_2, b_3, b_4 be positive integers such that

$$\frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \frac{1}{b_4} = 1.$$

Prove that at least one of the b_k's is even. Hint: clear the denominators.

(7) Let $c_1, c_2, c_3, c_4, \ldots, c_{2n}$ be $2n-$many positive integers such that

$$\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \frac{1}{c_4} + \frac{1}{c_4} + \cdots + \frac{1}{c_{2n}} = 1.$$

Prove that at least one of the c_k's is even. Hint: clear the denominators.

(8) A palindrome is an integer whose decimal expansion is symmetric, e.g. 1, 2, 11, 121, 15677651 (but not 010, 0110) are palindromes. Prove that there is no positive palindrome which is divisible by 10.
(9) Show that if a is rational and b is irrational, then $a + b$ is irrational.
(10) The product of 34 integers is equal to 1. Show that their sum cannot be 0.
(11) Prove that $\sqrt{3}$ is irrational.
(12) Let a, b, c be integers satisfying $a^2 + b^2 = c^2$. Show that abc must be even. (Harder problem: show that a or b must be even.)
(13) If a, b, c are odd integers, prove that $ax^2 + bx + c = 0$ does not have a solution x such that x is a rational number.
(14) Prove that if $3 \mid (a^2 + b^2)$, then $3 \mid a$ and $3 \mid b$. Hint: If $3 \nmid a$ and $3 \nmid b$, what are the possible remainders of a, b, a^2, and b^2 upon division by 3?