Errata for *Spectral Theory*

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Please email any additional corrections to *dborthw@emory.edu*.

Chapter 2

p. 10: 2nd paragraph
Missing power of $1/p$ in sentence two, should read: "Note that $\|\chi_A\|_p = \mu(A)^{1/p}$,..."

p. 17: Equation (2.17)
This polarization uses the conjugate convention for sesquilinear forms. For our convention it should read:

$$\langle u, v\rangle = \frac{1}{4}\Big(\|u+v\|^2 - \|u-v\|^2 + i\|u-iv\|^2 - i\|u+iv\|^2\Big).$$

p. 19: Equation (2.21)
Missing superscript on the left side: dx should be $d^n x$.

Chapter 3

p. 47: Definition 3.19
Should read: "An operator..."

p. 49: Lines 6 and 12
The domain $\mathcal{D}(A)$ should be $\mathcal{D}(T)$ (happens twice).

p. 51: Lemma 3.27
For the second statement of the lemma, we continue to assume that A is self-adjoint on $\mathcal{D}(A)$, and add the hypothesis that A is essentially self-adjoint on a sub-domain. Here is a corrected wording: "Furthermore, if A is essentially self-adjoint on a core domain contained in $\mathcal{D}(A)$, then $A+B$ is essentially self-adjoint on this core domain."

p. 55: Equation (3.25)
The semibounded condition should hold for $u \in \mathcal{D}(S)$, not the full Hilbert space.

p. 60: After (3.32)
The word "weak" is repeated.

Chapter 4

p. 70: Final equation
Extra "$u \in$" in the brackets. The equation should read

$$\mathcal{D}(M_f) := \big\{u \in L^2(X, d\mu) : fu \in L^2(X, d\mu)\big\}.$$

p. 71: Proof of Theorem 4.5
First line: "If $\lambda \in$ ess-range(f)..." (lower case f). Also, the final equation should read

$$\sigma(M_f) \subset \text{ess-range}(f),$$

i.e., the opposite direction to (4.3).

p. 76: Final paragraph
The range of index j should be $j \in \{1, \ldots, q\}$.

p. 78: Equation after (4.14)
The matrix should be

$$A(\theta) = \begin{pmatrix} 2\cos\theta_2 & 1 + e^{i\theta_1} \\ 1 + e^{-i\theta_1} & -2\cos\theta_2 \end{pmatrix}.$$

p. 79: Definition 4.6
The generic power series should start at $n = 0$. The sequence should be labeled $\{A_n\}_{n=0}^{\infty}$, and the lower summation limit should be $n = 0$.

p. 82: Afer (4.21)
Should read "for $n \in \mathbb{N}_0$."

p. 84: Line before Corollary 4.12
Should read "different values of the argument."

p. 84: Corollary 4.12
The power series expansion (4.26) only proves the identity locally. Here's the correct argument:

Proof. If z and w are sufficiently close to z_0, then the formula follows from (4.26) by manipulation of the geometric power series.

To prove the identity in general, note that $(T-z)^{-1}(T-z) = I$ on $\mathcal{D}(T)$ and $(T-z)(T-z)^{-1} = I$ on $\mathcal{H}$. Thus

$$\begin{aligned}(T-z)^{-1} - (T-w)^{-1} &= (T-z)^{-1}(T-w)(T-w)^{-1} - (T-z)^{-1}(T-z)(T-w)^{-1} \\ &= (T-z)^{-1}(z-w)(T-w)^{-1}.\end{aligned}$$

□

p. 84: Second resolvent identity
This formula should read,

$$(S-z)^{-1} - (T-z)^{-1} = (S-z)^{-1}(T-S)(T-z)^{-1},$$

both here and in Exercise 4.1

p. 85: Equation (4.28)
The right side should have a minus sign:

$$(T - z)^{-1} = -\sum_{n=0}^{\infty} z^{-n-1} T^n.$$

p. 85: Corollary 4.13
Technically, the assumption $\mathcal{H} \neq \{0\}$ should be included here.

p. 86: Equation after (4.29)
The formula should read:

$$F(z) = -\frac{1}{z}\left(T - \frac{1}{z}\right)^{-1}.$$

p. 91: Fourth equation
This should read: By (4.31),

$$(I - F(z))Q(z)^{-1}v = 0.$$

p. 91: Final paragraph of the proof of Thm. 4.19
The specification of A and B is a bit unclear. Here is a cleaner version:

$$A := \big\{z \in \Omega : I - F(\cdot) \text{ fails to be invertible at all points in some neighborhood of } z\big\}$$

and

$$B := \big\{z \in \Omega : I - F(\cdot) \text{ is invertible in a neighborhood of } z \text{ except possibly on a discrete set}\big\}.$$

p. 92: Proof of Thm. 4.21
In the second sentence of the second paragraph, W should be $\mathcal{W}$.

p. 96: Exercise 4.1
The second resolvent identity should read

$$(S - z)^{-1} - (T - z)^{-1} = (S - z)^{-1}(T - S)(T - z)^{-1}.$$

Chapter 5

p. 107: Proof of Thm. 5.5
The measure ν in the final equation should perhaps have been defined more explicitly. A subset $E \subset Y$ consists of a collection of subsets $E_k \subset \mathbb{S}$. The measure is given by

$$\nu(E) := \sum_k \nu_k(E_k).$$

Since the measures ν_k are finite, the measure ν is σ-finite.

p. 109: Proof of Thm. 5.6
First sentence should read "let U be the corresponding..."

p. 110: Equation (5.15)
For consistency, change $(1+U)$ to $(I+U)$ in the final line.

p. 111: Example 5.8
In the second equation, π should appear in the denominator:

$$\begin{aligned}\langle v, f(U)v\rangle &= \frac{1}{\pi}\int_{\mathbb{R}} f(\gamma(x))\,\frac{1}{x^2+1}\,dx \\ &= \int_0^{2\pi} f(e^{i\theta})\,\frac{d\theta}{2\pi}.\end{aligned}$$

p. 112: Thm. 5.9
Uniqueness of the map requires some extra condition that will guarantee that for $h_z(x) := (x-z)^{-1}$ with z strictly complex,

$$h_z(A) = (A-z)^{-1}.$$

This is not implied by (a), because the function $f(x) = x - z$ is not in $\mathcal{B}_{\mathrm{b}}(\mathbb{R})$. The easiest fix is modify the final sentence of the theorem to read: "Moreover, (5.16) gives the unique map $\mathcal{B}_{\mathrm{b}}(\mathbb{R}) \to \mathcal{L}(\mathcal{H})$ satisfying these conditions and for which h_z maps to $(A-z)^{-1}$ for $h_z(x) := (x-z)^{-1}$ with z strictly complex."

p. 113: Proof of Thm. 5.9
After (5.19) should read "The second identity..."

Since f is complex valued, the equation after (5.19) should read

$$\mu\Big\{x \in X :\ |f\circ\alpha(x) - f(\lambda)| < \varepsilon\Big\} > 0.$$

Also, several lines after (5.19), $\sigma_{\mathrm{ess}}(f\circ\alpha)$ should be replaced by ess-range$(f\circ\alpha)$. The correct sentence is "Hence, $f(\lambda) \in$ ess-range$(f\circ\alpha)$."

p. 114: Proof of Thm. 5.9
The functions h_{z_j} in the second sentence were not defined. This is taken care of by the modification of the theorem statement as described on p. 112.

Furthermore, the application of Stone-Weierstrass requires a restriction to functions that vanish at infinity. To avoid extra notation, we can replace $C_b(\mathbb{R})$ by $C_0(\mathbb{R})$, where the 0 denotes compact support. In the following paragraph, the claim should be that the characteristic function of an interval can be approximated pointwise by a function in $C_0(\mathbb{R})$.

p. 115: Proof of Thm. 5.10
The claim that the integral expression on the right side of (5.21) equals $f_\varepsilon(A)$ should be justified. The integral in (5.21) defines an operator

$$B_\varepsilon := \frac{1}{2\pi i}\int_{-\infty}^{\infty} f(\lambda)\Big[(A-\lambda-i\varepsilon)^{-1} - (A-\lambda+i\varepsilon)^{-1}\Big]\,d\lambda.$$

This can be interpreted in the weak sense described in §4.2.1, as the unique operator for which

$$\langle u, B_\varepsilon v\rangle = \frac{1}{2\pi i}\int_{-\infty}^{\infty} f(\lambda)\Big\langle u, \Big[(A-\lambda-i\varepsilon)^{-1} - (A-\lambda+i\varepsilon)^{-1}\Big]v\Big\rangle\, d\lambda,$$

for all $u, v \in \mathcal{H}$. Using the unitary transformation Q provided by the spectral theorem (Theorem 5.6), this can be written

$$\langle u, B_\varepsilon v\rangle = \frac{1}{2\pi i}\int_{-\infty}^{\infty} f(\lambda)\Big\langle Q^{-1}u, \Big[(x-\lambda-i\varepsilon)^{-1} - (x-\lambda+i\varepsilon)^{-1}\Big]Q^{-1}v\Big\rangle\, d\lambda,$$

where the inner product now takes place in $L^2(X, d\nu)$. Since f is integrable and the expression in brackets is bounded, Fubini's theorem allows us to take the λ integral first, yielding

$$\begin{aligned}\langle u, B_\varepsilon v\rangle &= \big\langle Q^{-1}u, f_\varepsilon(x)Q^{-1}v\big\rangle \\ &= \langle u, f_\varepsilon(A)v\rangle.\end{aligned}$$

This proves that $B_\varepsilon = f_\varepsilon(A)$.

p. 115: Final equation
The Qs are reversed, should read:

$$\Pi_E = Q\chi_{\alpha^{-1}\{E\}}Q^{-1}.$$

p. 116: Proof of Thm 5.11
Missing brackets in the second paragraph: the function $Q^{-1}\phi$ has support on $\{\alpha = \lambda\}$.

p. 117: First equation
For the sake of linearity, the inner product on the right-hand side should be switched:

$$\Pi_{(\lambda-\varepsilon,\lambda+\varepsilon)}u = \sum_{j=1}^{k}\langle e_j, u\rangle e_j.$$

p. 117: Third equation
The norms should be squared:

$$\begin{aligned}\|(A-\lambda)u_n\|^2 &= \big\|(A-\lambda)(1-\Pi_{(\lambda-\varepsilon,\lambda+\varepsilon)})u_n\big\|^2 + \big\|(A-\lambda)\Pi_{(\lambda-\varepsilon,\lambda+\varepsilon)}u_n\big\|^2 \\ &\geq \varepsilon^2\big\|(1-\Pi_{(\lambda-\varepsilon,\lambda+\varepsilon)})u_n\big\|^2 - \varepsilon^2\big\|\Pi_{(\lambda-\varepsilon,\lambda+\varepsilon)}u_n\big\|^2\end{aligned}$$

p. 121: Problem 5.2(b)
Should start "Prove that the spectrum..." (remove the first [if]).

Chapter 6

p. 133: First equation
The γ is erroneous. The first equation should read

$$|\langle f, v\rangle| \leq \|f\|\|v\| \leq \|f\|\|v\|_{H^1},$$

p. 136: Proof of Thm 6.8
The first sentence of the second paragraph is mixed up. This paragraph should read as follows:
If $u \in \mathcal{D}(-\Delta_{\mathrm{D}})$, then by (6.13) we have

$$\|u\|_{H^1}^2 = \langle u, (-\Delta + 1)u \rangle. \tag{1}$$

By Cauchy-Schwarz and the fact that $\|u\| \leq \|u\|_{H^1}$, this implies that

$$\|u\|_{H^1} \leq \|(-\Delta + 1)u\|.$$

This shows that $(-\Delta_{\mathrm{D}} + 1)^{-1}$ is bounded as a map $L^2(\Omega) \to H_0^1(\Omega)$. Therefore $(-\Delta_{\mathrm{D}} + 1)^{-1}$ is compact as a map $L^2(\Omega) \to L^2(\Omega)$ by Theorem 6.9.

p. 140: Lemma 6.14
Second sentence should start "A function u..."

p. 144: Corollary 6.16
The assumption on ψ should be $\psi \in H^1(\Omega)$.

p. 145: Proof of Lemma 6.17
The wording of the first sentence suggests that (6.31) is an assumption rather than a goal. Replace this sentence with: "In order for $G(ik; \cdot)$ to satisfy the first line of (6.31), it should satisfy

$$(-\Delta - k^2)G(ik; |x|) = 0$$

for $x \neq 0$."

p. 147: Line 5
Period should be a comma: "More generally, one can..."

p. 150: Proof of Thm. 6.20
Third sentence should read "By Lemma 6.12, these functions can be interpreted..."

p. 151: Equation (6.48)
The σ in the brackets should be $\sigma(-\Delta_{\mathrm{D}})$.

p. 151: Equation (6.49)
These inequalities are backwards. The equation should read:

$$N_{\mathcal{R}_1}(t) \leq N_\Omega(t) \leq N_{\mathcal{R}_2}(t).$$

p. 160: Thm. 6.30
For clarification, add a parenthetical remark to the last line: "as $h \to 0$ (meaning $O(h^N)$ for any $N > 0$),..."

p. 168: Equation after (6.86)
The definition should read

$$f_\delta(x) := \begin{cases} 1, & x \in [0,1], \\ 1-(x-1)/\delta, & x \in (1, 1+\delta), \\ 0, & x \geq 1+\delta. \end{cases}$$

p. 168: Last three equations
The μ_t on the left side of the last three equations should be ν_t, as defined at the beginning of the proof.

p. 170: Sentence after (6.92)
The a should be capitalized in $A\nu^{-1}s^\nu$ and in the following equation. Also, the change of variable from s to u makes this the reference to (6.91) unclear. Here is a clean version of the two sentences following (6.92):

For convenience, set $\nu := m - n/2$ and rewrite (6.91) in the form

$$s^{-\nu}\int_0^s y^{m-1} f(y)\,dy = A\nu^{-1} + o(1).$$

This means that $\varepsilon > 0$ we can choose $c_\varepsilon > 0$ so that

$$\left| s^{-\nu}\int_0^s y^{m-1} f(y)\,dy - A\nu^{-1} \right| < \varepsilon$$

for $s \leq c_\varepsilon$.

p. 172: Proof of Thm 6.34
The definition of $\psi_1^\pm$ should read

$$\psi_1^\pm(x) := \max\{\pm\psi_1(x), 0\}.$$

p. 174: First paragraph of §6.7
Replace "a disk in $\mathbb{R}^2$ of radius r" "the unit disk in $\mathbb{R}^2$"

p. 175: Thm. 6.26
Missing "in" in the first sentence: ".. bounded open set in $\mathbb{R}^n$"

p. 177: Proof of Thm 6.36
The first sentence should read: "Let ϕ_1 be the eigenfunction...". (Since λ_1 is simple, ϕ_1 is uniquely defined.) In the 4th paragraph of the proof, the range of t should be $t \in [0, T]$.

p. 178: After (6.107)
The function ψ should not have a subscript: "and $\{\psi = 0\}$ is smooth"

Chapter 7

p. 185: Proof of Thm 7.1

In the first line of the second paragraph, the argument that $\psi u \in \mathcal{D}(A^*)$ is easy only if we assume that $u \subset H^1_{\mathrm{loc}}$. While that was true for the Dirichlet Laplacian, this assumption is not valid here.

To fix the argument, consider a bounded open set $\Omega \subset \mathbb{R}^n$. For $u \in \mathcal{D}(A^*)$ and $\phi \in C_0^\infty(\Omega)$ we have

$$|\langle u, -\Delta\phi\rangle| \le \Big(\|A^*u\| + \sup_\Omega V\Big)\|\phi\|.$$

By the Riesz lemma this implies that $(-\Delta u)|_\Omega$ exists (in the weak sense) in $L^2(\Omega)$. A standard interior elliptic regularity result, such as Thm. 8.12 of Rudin [79] (*Functional Analysis*), then implies that $u|_\Omega \in H^2_{\mathrm{loc}}(\Omega)$. Since Ω was arbitrary this gives $u \in H^2_{\mathrm{loc}}(\mathbb{R}^n)$.

Note that Thm. 8.12 of Rudin differs from the elliptic regularity result included in §A.4, in that the starting assumption is $u \in L^2_{\mathrm{loc}}$ rather than H^1_{loc}. This means that applying a cutoff $\chi \in C_0^\infty(\Omega)$ takes us to $\Delta(\chi u) \in H^{-1}(\Omega)$. I was trying to avoid this complication, because H^{-1} was not otherwise defined in this text.

p. 188: Proof of Thm. 7.3

After the first equation in the proof, the domains should be "$u \in \mathcal{H}_Q$ and $v \in \mathcal{D}(A)$"

p. 193: First eq. after Fig. 7.1

The γ should be ω:

$$U_\omega f(x) := \omega^{\frac{1}{4}} f(\omega^{\frac{1}{2}} x).$$

p. 194: Equation (7.26)

The assumption should be $u \in \mathcal{H}$, rather than $\mathcal{D}(A)$.

p. 195: Proof of Thm 7.7

In last part of the proof, B was mistakenly assumed to be closed. Here is a clean version of the final two paragraphs:

Now assume that A is merely essential self-adjoint. If $u \in \mathcal{D}(\overline{A})$, then there exists a sequence $u_n \to u$ with $u_n \in \mathcal{D}(A)$, such that Au_n converges to $\overline{A}u$. By the assumption (7.24), the sequence Bu_n also converges, so that $u \in \mathcal{D}(\overline{B})$. By continuity, we can extend (7.24) to

$$\big\|\overline{B}u\big\| \le \alpha\big\|\overline{A}u\big\| + \beta\|u\| \tag{2}$$

for all $u \in \mathcal{D}(\overline{A})$. By the first part of the proof, this implies that $\overline{A} + \overline{B}$ is self-adjoint on the domain $\mathcal{D}(\overline{A})$.

It remains to check that $\overline{A+B} = \overline{A} + \overline{B}$. Since $\overline{A} + \overline{B}$ is a closed extension of $A + B$, we have $\overline{A+B} \subset \overline{A} + \overline{B}$. On the other hand, the assumption (7.24) gives

$$\|(A+B)u\| \le (\alpha+1)\|Au\| + \beta\|u\|.$$

For $u \in \mathcal{D}(\overline{A})$ this implies that $u \in \mathcal{D}(\overline{A+B})$ and that $(\overline{A+B})u = (\overline{A} + \overline{B})u$. In other words,

$$\overline{A} + \overline{B} \subset \overline{A+B}.$$

We conclude that $\overline{A+B}$ is self-adjoint on $\mathcal{D}(A)$.

p. 196: Equation (7.29)
Equation (7.29) is related to (7.28) by square root, so the power of b should be halved:

$$\|u\|_\infty \leq C b^{\frac{n}{2}-2} \left\|(-\Delta + b^2)u\right\|.$$

In the next equation the same power of b appears and the second line should be an inequality:

$$\begin{aligned} \|Vu\| &\leq \|V\| \|u\|_\infty \\ &\leq C b^{\frac{n}{2}-2} \|V\| \Big(\|\Delta u\| + b^2 \|u\| \Big), \end{aligned}$$

p. 197: Second paragraph
Misplaced circumflex in the equation block, $f(\hat{\xi})$ should be $\hat{f}(\xi)$.

p. 198: First equation
The sign is wrong here:

$$(A-z)^{-1}(A-\lambda) = I + (z-\lambda)(A-z)^{-1}.$$

This sign mistake is repeated in (7.32), (7.33), and in the equation after (7.34).

p. 198: Sentence containing (7.35)
The right side is missing from the first equation: rewriting (7.33) gives

$$\lim_{k\to\infty} \|u_k - (z-\lambda)w_k\| = 0.$$

The second citation "by (7.33)" after (7.35) should be dropped.
After (7.35), the first sentence should say $\sigma \notin \sigma_{\mathrm{ess}}(B)$.

p. 199: Second equation
Missing reciprocal, should read

$$\liminf_{k\to\infty} \|(B-\lambda)w_k\| \geq \frac{\varepsilon}{|z-\lambda|}.$$

p. 199: Final line
The limit should be $\sigma \to \infty$.

p. 200: Proof of Theorem 7.12
The limit in (7.39) should be $\sigma \to \infty$. In the final sentence of the proof, Theorem 7.11 should be cited instead of Theorem 5.14.

p. 200: Corollary 7.13
In the first sentence of the corollary, as well as at the start of the preceding paragraph, the assumption should be "self-adjoint on a domain contained in $H^1(\mathbb{R}^n)$".

p. 202: Proof of Lemma 7.15
In the first line of the final equation block the integrand should be $V(x)^2 G_n(|x-y|)^2$.

p. 203: Second equation
Missing absolute value on the right, should be

$$\left\|(1-\chi_n)V(-\Delta+1)^{-1}\right\| \leq \sup_{|x|\geq n} |V(x)|.$$

p. 205: Equation (7.45)
The range for m should be $\{-l, \ldots, l\}$

p. 206: First equation
A factor of r^2 is missing in the h'' term:

$$r^2 h'' + 2rh' + \big(r + \lambda r^2 - l(l+1)\big)h = 0.$$

Also, the first sentence second paragraph should read "...extract the asymptotic behavior as $r \to \infty$," not $r \to 0$.

p. 206: Final line
The equation should end with a period rather than a comma.

p. 208: Equation after (7.51)
The right side should be $[0, c)$, since no argument was given to rule out a zero eigenvalue.

p. 211: Equation after (7.58)
The max should be taken over $u \in W\backslash\{0\}$.

p. 211: Equation (7.61)
The γ was switched to a δ here. Should be γ for the rest of the proof.

Chapter 8

p. 226: After (8.2)
In the final sentence of the paragraph, should read "but we will not consider those cases here."

p. 227: Paragraph before Lemma 8.1
First sentence should read: "we at least know that L is a symmetric operator on the space..."

p. 227: After (8.7)
Should read "..vertex v_k such that $\phi_2(v_k)$ has sign opposite..."

p. 232: Examples 8.6 and 8.7
The number of edges was notated inconsistently in these two examples. Each instance of k should be replaced by m, the number of edges.

p. 233: First equation
The sum should be $\sum_{j=1}^{m}$.

p. 233: Example 8.7
In the third sentence, should refer to "$m-1$ independent eigenfunctions"

p. 235: Proof of Cor. 8.9
The final sentence of the first paragraph should read: "...the fact that $\tilde{\lambda}_k$ is given by..."

Chapter 9

p. 246: last full paragraph
Missing "if" in "if and only if"

p. 249: Equation after (9.3)
The left side should be written $\xi(v)$, since the dot convention for the pairing is not used elsewhere.

p. 253: Second paragraph
Missing period after the first sentence.

p. 260: Proof of Thm 9.15
In the second equation block, the equality in the first line should be

$$\ell(\gamma) = \int_0^1 |\dot{\gamma}|\, dt.$$

p. 269: Sentence containing (9.39)
The partition of unity should be indexed as $\{\chi_j\}_{j=1}^q$.

p. 270: Thm. 9.26
Strike the repeated "for $m > k + n/2$" at the end of the statement.

p. 272: Proof of Thm. 9.29
In the final equation block, the second line should be $2\langle v, \Delta v\rangle$.

p. 276: Equation (9.53)
Missing ∂_r in the final term on the right:

$$\Delta\psi = \partial_r^2\psi + (n-1)r^{-1}\partial_r\psi + \partial_r(\log\varphi_y)\partial_r\psi.$$

p. 277: Equation before (9.55)

Wrong sign in the $\partial_j u_j$ terms:

$$
\begin{aligned}
&(\partial_t - \Delta_x) \sum_{j=0}^{k} t^j u_j \Psi(t;r) \\
&\quad = \sum_{j=0}^{k} \Big[j u_j t^{j-1} + \frac{r}{2t} \partial_r (\log \varphi_y) u_j t^j - t^j \Delta_x u_j + \frac{r}{t} (\partial_r u_j) t^j \Big] \Psi \\
&\quad = \Big[\frac{r}{2} \partial_r (\log \varphi_y) u_0 + r \partial_r u_0 \Big] t^{-1} \Psi \\
&\qquad + \sum_{j=1}^{k} \Big[j u_j + \frac{r}{2} \partial_r (\log \varphi_y) u_j - \Delta_x u_{j-1} + r \partial_r u_j \Big] t^{j-1} \Psi - (\Delta_x u_k) t^k \Psi.
\end{aligned}
$$

p. 285: Second paragraph

The second sentence should start "Then $\Delta^l u_0$ and $\Delta^l v_0$..."

p. 286: Proof of Thm. 9.38

The second paragraph should start "Given $p \in \overline{\Omega}$,..."

p. 289: Proof of Thm. 9.40

In the equation after (9.82), v is missing from the second line:

$$
\begin{aligned}
f^{(k)}(t) &= \Big\langle w, (iA)^k U(t) v \Big\rangle \\
&= i^k \Big\langle (A^k)^* w, U(t) v \Big\rangle \\
&= \mp i^{k+1} f(t).
\end{aligned}
$$

The final equation of this proof has an extra w, should be

$$
\ker\Big((A^k)^* \pm i \Big) = \{0\}.
$$

p. 289: Proof of Thm. 9.41

In the third sentence, the index of K should not be restricted to an integer. Starting with this sentence, it should read: "By the Heine-Borel property, the set

$$
K_r := \{ p \in M : \operatorname{dist}(p, K) \leq r \}
$$

is compact for $r \geq 0$. For each $j \in \mathbb{N}$,.."

p. 290: After (9.84)

At the end of the paragraph this should should be a direct sum $\mathcal{W} \oplus L^2(M)$. This typo is repeated in the paragraph after Thm. 9.42.

p. 292: Thm. 9.43

First sentence should end with the singular "manifold."

p. 295: Equation (9.94)
Should end with a period.

p. 297: First equation
This should read

$$\chi_m(x) = \begin{cases} 1, & \rho(x) \le m, \\ 0, & \rho(x) \ge m+1. \end{cases}$$

$\boxed{\text{Appendix}}$

p. 307: Product measure
In standard definition of product measure, $\mathcal{M}$ is defined as the σ-algebra generated by the measurable rectangles in $X_1 \times X_2$. What is described here, using Carathéodory's condition, is the completion of the product measure.

p. 308: Sentence before Thm. A.7
Missing word: "It turns out that..."

p. 313: Equation (A.14)
The assumption for this claim should be $0 \le f \le 1$, and the equation should read

$$\mu\{f = 1\} \le \beta(f) \le \mu(\operatorname{supp} f).$$

p. 322: Equations (A.34) and (A.37)
In both of these equations, the integral on the right side should have a factor of $(2\pi)^{-\frac{n}{2}}$.

p. 324: Lemma A.21
Index should be j in the equation:

$$\|\partial_j^h u\|_{L^2} \le \|\partial_j u\|_{L^2}.$$

p. 326: Paragraph after (A.45)
The second sentence should start: "Assuming that...". In the final equation of this paragraph the operator L should be $-\Delta$.

p. 327: Equation (A.47)
Second line should end with $\|\nabla v\|$.

p. 327: Final paragraph
In correct plural, should be "there exist functions f_j...". The first sentence should end with $l \to \infty$.

p. 329: Second equation
The left side should be $\|\Delta(D^\alpha u)\|$.

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