Errata for *Introduction to Partial Differential Equations*

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December 2017

Note, these errata appeared in the first printing. Both online and print editions were updated in Spring 2018, so these issues should not appear in more recent copies of the book.

Please email any additional corrections to `davidb@mathcs.emory.edu`.

Chapter 2

p. 11: Equation (2.2)
The expansion is missing some minus signs:

$$e^{i\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right) + i\left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \dots\right)$$

p. 17: 2nd formula from the bottom
The sum should start at $j = 0$:

$$\boldsymbol{u}_k(t) = \sum_{j=0}^{k} \frac{t^j}{j!} \begin{pmatrix} 0 & 1 \\ -\kappa^2 & 0 \end{pmatrix}^j \begin{pmatrix} a \\ b \end{pmatrix}$$

p. 17: Last equation
Missing factor of $1/\kappa$ in the second term:

$$\begin{aligned}\boldsymbol{w}(t) = &\left[1 - \frac{(\kappa t)^2}{2!} + \frac{(\kappa t)^4}{4!} - \dots\right] \begin{pmatrix} a \\ b \end{pmatrix} \\ &+ \frac{1}{\kappa}\left[\kappa t - \frac{(\kappa t)^3}{3!} + \frac{(\kappa t)^5}{5!} - \dots\right] \begin{pmatrix} b \\ -\kappa^2 a \end{pmatrix}.\end{aligned}$$

p. 18: First equation
Missing factor of $1/\kappa$ in the second term:

$$y(t) = a\cos(\kappa t) + \frac{b}{\kappa}\sin(\kappa t).$$

p. 21: Equation (2.11)

The variable r should be ρ, and the limit of integration is missing:

$$\int_{\mathbb{B}^3} \nabla \cdot \boldsymbol{F}\, d^3\boldsymbol{x} = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-\rho^2}}^{\sqrt{1-\rho^2}} \frac{\partial f}{\partial z}\, \rho\, dz\, d\rho\, d\theta$$
$$= \int_0^{2\pi} \int_0^1 \Big[f\Big(\rho\cos\theta, \rho\sin\theta, \sqrt{1-\rho^2}\Big)$$
$$- f\Big(\rho\cos\theta, \rho\sin\theta, -\sqrt{1-\rho^2}\Big)\Big]\, \rho\, d\rho\, d\theta.$$

Chapter 3

p. 38: Equation (3.30)

The final x should be x_0:

$$x(t) = \begin{cases} x_0 - t, & x_0 \le 0, \\ x_0 + (2x_0 - 1)t, & 0 < x_0 < 1, \\ x_0 + t, & x_0 \ge 1. \end{cases}$$

p. 43: Burgers' equation

The apostrophe should follow the "s", as the equation is named for Dutch physicist Johannes Burgers.

Chapter 4

p. 46: 2nd equation

The numerator in the expression for $\sin\beta_j$ has the wrong sign:

$$\sin\alpha_j \approx \frac{u(t, x_{j-1}) - u(t, x_j)}{\Delta x}, \quad \sin\beta_j \approx \frac{u(t, x_{j+1}) - u(t, x_j)}{\Delta x}.$$

p. 50–51: Huygens' principle

The argument presented in Theorem 4.3 is valid for the g term in (4.8), but not for not the h term. The mistake is repeated in Figure 4.4, which shows only the support of the g term. Here is a corrected version of this portion of the text:

> **Theorem 4.3 (Huygens' principle in dimension one).** *Suppose u solves the wave equation* (4.5) *for $t \geq 0$, $x \in \mathbb{R}$, with initial data given by* (4.7). *If the functions g, h are supported in a bounded interval $[a, b]$, then*
>
> $$\operatorname{supp} u \subset \Big\{(t, x) \in \mathbb{R}^+ \times \mathbb{R};\ x \in [a - ct, b + ct]\Big\}.$$
>
> *Proof.* Consider the components of the solution (4.8). The g term will vanish unless $x \pm ct \in [a, b]$. The support of this term is thus restricted to $x \in [a - ct, b - ct]$ or $x \in [a + ct, b + ct]$. As for the h term, the integral over τ will vanish unless the interval $[x - ct, x + ct]$ intersects $[a, b]$, which occurs only when $x \in [a - ct, b + ct]$. □
>
> The restriction of support described in Theorem 4.3 is illustrated in Figure 4.4. The g term contributes only in the regions shown in blue, but the h term may contribute throughout the full support region. However, the solution is constant (equal to $\int_a^b h(\tau)\, d\tau$) when $[a, b]$ is contained in $[x - ct, x + ct]$. This constant region shown in purple in Figure 4.4.

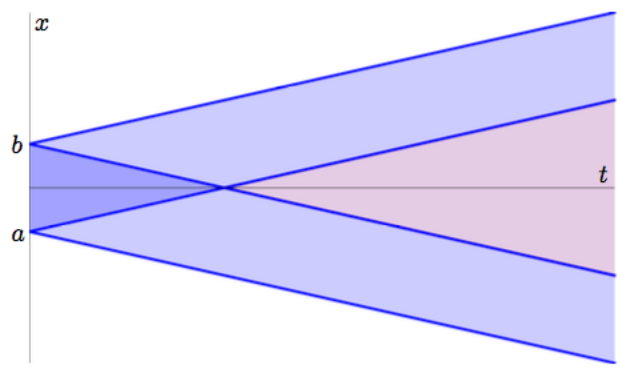

Fig. 4.4. Support of a wave solution with initial data in a bounded interval.

p. 52: Theorem 4.5

The condition that g vanishes at the endpoints does not guarantee that the extension of g is C^2. The formula (4.8) yields a classical solution only under this extra hypothesis. Furthermore, although the solution is indeed unique (as proved later in Corollary 4.13), that fact is not part of the argument here. Here is a corrected statement of the theorem:

> **Theorem 4.5.** *The wave equation* (4.5) *on* $[0, \ell]$, *with Dirichlet boundary conditions and satisfying the initial conditions* (4.16), *admits a solution of the form* (4.8) *only if the initial data admit extensions to* $\mathbb{R}$ *as odd,* 2ℓ*-periodic functions, with* $g \in C^2(\mathbb{R})$ *and* $h \in C^1(\mathbb{R})$.

p. 53: 2nd paragraph and Figure 4.6
This paragraph and figure should refer to u_- rather than u_+, and the equation should read:

$$h(x) = \frac{\partial}{\partial t} u_-(x+t)\big|_{t=0} = \frac{du_-}{dx}(x).$$

p. 61: Lemma 4.9
Should read "For $f \in C^2(\mathbb{R}^3)$,..."

p. 65: strict Huygens principle
The mistake from Theorem 4.3 is unfortunately repeated here. The final sentence of the first paragraph should read:

> The strict Huygens' principle holds in every odd dimension greater than 1, but fails in even dimensions, as we will illustrate below.

Chapter 6

p. 98: Second equation from the bottom
T_0 and T_1 are switched here; it should read

$$u_0(x) := T_0\Big(1 - \frac{x}{\ell}\Big) + T_1 \frac{x}{\ell}$$

p. 99: Example 6.1
The bounded interval was intended to be $[0, \ell]$, with the boundary condition $u(0) = u(\ell) = 0$. Setting $\ell = \pi$ would allow some cancellation in the formulas.

Chapter 11

p. 237: Exercise 11.2
The function $\log r$ is not contained in H^1 in dimension 2. The problem should have been stated on the domain $\mathbb{B} = \{r < 1\} \subset \mathbb{R}^3$, so that $\log r \in H^1_0(\mathbb{B})$. Furthermore, $r := |\boldsymbol{x}|$ (and not $|\boldsymbol{x}|^2$). The function $u = \log r$ is a weak solution of $Lu = f$ with $f = 1$, so this still provides an example where f and $\partial\mathbb{B}$ are smooth but u is not.

Acknowledgments: Thanks for Harald Hanche-Olsen and Morten Grønbech for pointing out corrections to the text.