Errata for *Introduction to Partial Differential Equations*

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This is the errata list for the updated text as of Spring 2018. A number of mistakes from the first publication were corrected in the print and online versions at that point. Please email any additional corrections to `davidb@mathcs.emory.edu`.

Chapter 1

p. 4: Equations (1.8) and (1.9)
There is a typo in the notation for the second partials. In both equations this term should be

$$-\sum_{i,j=1}^{n} a_{ij}(\boldsymbol{x})\frac{\partial^2 u}{\partial x_i \partial x_j}.$$

Chapter 2

p. 15: Final equation
This should have read

$$\int \frac{dy}{g(y)} = \int h(t)\,dt.$$

Chapter 4

p. 49: Equations after (4.13)
The use of $\partial g/\partial x$ is potentially confusing, since g depends on a single variable. This should be replaced by g' in the two unnumbered formulas.

p. 49: Equation (4.14)
Should be $u(t, x)$ to be consistent with the usage elsewhere.

p. 52: Theorem 4.5
The hypotheses for this theorem are unclear, because (4.8) does not make sense unless g and h are defined on all of $\mathbb{R}$. This should be clarified into two separate statements:

(1) If g and h admit extensions to 2ℓ-periodic functions in $C^2(\mathbb{R})$ and $C^1(\mathbb{R})$, respectively, then the initial value problem (4.15, 4.16) admits a solution of the form (4.8).
(2) Suppose $u(t, x)$ solves the wave equation on $\mathbb{R} \times \mathbb{R}$. If the restriction of u to $x \in [0, \ell]$ satisfies Dirichlet boundary conditions for all t, then $u(t, x)$ can be written in the form (4.8) where g and h are odd and 2ℓ-periodic.

The argument given in the text proves (1), but the proof of (2) requires some extra detail to separate out the symmetry properties of g and h. Perhaps the easiest approach is to invoke the formula (4.14):

$$u(t, x) = u_+(x - ct) + u_-(x + ct).$$

The condition $u(t,0) = 0$ implies $u_+(x) = -u_-(-x)$, and then $u(t,\ell) = 0$ implies that

$$u_-(ct+\ell) = u_-(ct-\ell)$$

for all t, so that u_- is 2ℓ-periodic. From

$$u(t,x) = -u_-(ct-x) + u_-(ct+x)$$

we can then deduce the symmetries of g and h.

p. 55: Theorem 4.7
The function $f \in C^1(\mathbb{R} \times \mathbb{R})$, since the driving term is time-dependent.

p. 57: Sentence after (4.28)
The function $\sin(\omega_0 x)$ is 2ℓ-periodic.

p. 58: Figure 4.11
The periods should be labelled $2\pi/\omega$ and $2\pi/\omega_0$. This mistake also occurs in the accompanying text.

p. 62: First equation
The factor of $1/4\pi$ should either appear on both sides or neither.

Chapter 5

p. 77: Theorem 5.2
The condition should read "if and only if $\lambda = \lambda_n$ for $n \in \mathbb{N}$, where $\lambda_n := \frac{\pi^2 n^2}{\ell^2}$."

p. 84: Equation (5.20)
Should be λ, not λ^2.

p. 93: Problem 5.2
Frequencies are related to eigenvalues by $\omega = \sqrt{\lambda}$, if physical constants are omitted. The problem include a reference here to §5.2.

p. 93: Problem 5.4
Repeated "that."

Chapter 6

p. 99: Example 6.1
This example assumes that $k/c\rho = 1$ in (6.4).

p. 101: After equation (6.11)
The rescaling should read $(t,x) \mapsto (\lambda^2 t, \lambda x)$.

p. 102: Equation (6.12)
The $x = 0$ case should be C_2 instead of 0.

p. 103: First equation
Misplaced prime on the right. The first line should read:

$$\int_{-\infty}^{\infty} \varphi'(z)\Theta(x-z)\,dz = \int_{-\infty}^{x} \varphi'(z)\,dx$$

p. 104: Equation (6.16)
A minus sign is missing from the exponent: $e^{-|x|^2/4t}$.

p. 105: First equation
Missing factor of 1/4 in the exponent. The first line should read:

$$u(t,\boldsymbol{x}) = (4\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} e^{-|\boldsymbol{w}|^2/4} g\big(\boldsymbol{x} + t^{\frac{1}{2}}\boldsymbol{w}\big) d^n\boldsymbol{w}.$$

p. 108: Equation (6.28)
The second term should have f instead of $\partial f/\partial t$, so that the formula reads

$$\begin{aligned}\frac{\partial u}{\partial t}(t,\boldsymbol{x}) = &\int_0^t \int_{\mathbb{R}^n} H_s(\boldsymbol{y}) \frac{\partial f}{\partial t}(t-s,\boldsymbol{x}-\boldsymbol{y})\,d^n\boldsymbol{y}\,ds \\ &+ \int_{\mathbb{R}^n} H_t(\boldsymbol{y}) f(0,\boldsymbol{x}-\boldsymbol{y})\,d^n\boldsymbol{y},\end{aligned}$$

p. 108: Equation (6.30)
It should be pointed out that this holds independently of ε, for $\varepsilon > 0$.

p. 109: Exercise 6.2
The boundary condition is incompatible with the initial condition. It should be

$$u(t,0) = A\sin(\omega t).$$

p. 112: Equation (7.1)
The complex inner product should perhaps have called Hermitian rather than Euclidean.

Chapter 8

p. 136: Exercise 8.2
In the formula for $c_k[h]$, the factor of $(-1)^k$ is irrelevant because $\sin(\pi k/2) = 0$ if k is odd.

p. 142: Sentence after (8.32)
The intended reference for pointwise convergence was Theorem 8.3, not 8.10.

p. 142: Final paragraph
Should have included a remark that the uniform convergence of Theorem 8.5 implies L^2 convergence.

p. 144: Corollary 8.7
The norm on the right-hand side of the identity should be squared.

p. 144: Equation (8.36)
To prove this, the combination $f+g$ will only give $\mathrm{Re}\langle f, g\rangle$. One must also consider something like $f + ig$.

p. 146: Paragraph before Theorem 8.12
The equation (8.40) implies uniform convergence not by Theorem 8.5, rather by equation (8.33) from the end of its proof.

p. 147: Equation (8.42)
In (8.42) and the unnumbered equation preceding it, t should be replaced by y.

Chapter 9

p. 158: Sentence after (9.8)
Replace f by g.

p. 166: Equation (9.25)
This is in fact an equality.

p. 171: First line of the proof of Theorem 9.8
Should read $(t_0, \boldsymbol{x}_0) \in (0,T) \times \Omega$, not $\subset$.

p. 175: Exercise 9.3
In (a), the constant c actually depends on f and n but not on R. The constant C in (b) depends on n as well as R.

Chapter 10

p. 178: Equation (10.4)
The dimension is meant to be n here:

$$\int_\Omega u \frac{\partial \psi}{\partial x_j}\, d^n\boldsymbol{x} = -\int_\Omega f\psi\, d^n\boldsymbol{x}$$

for all $\psi \in C^\infty_{\mathrm{cpt}}(\mathbb{R}^n)$.

p. 179: Lemma 10.1
This proof is incorrect: L^1_{loc} is not contained in L^2_{loc}. This Lemma requires some basic measure theory arguments, which I was trying to avoid since measure theory is not assumed as a prerequisite. The standard proof would be to introduce a function $\psi \in C^\infty_{\mathrm{cpt}}(\mathbb{R}^n)$ with

$$\int_{\mathbb{R}^n} \psi\, d^n\boldsymbol{x} = 1.$$

For $\delta > 0$, defined the rescaled function $\psi_\delta(x) := \delta^{-n}\psi(x/\delta)$. Then $f * \psi_\delta = 0$ for all $\delta > 0$, by hypothesis. A 'mollification' argument shows that $f * \psi_\delta \to f$ as $\delta \to 0$, both in L^1_{loc} and also pointwise almost everywhere.

p. 188: Equation (10.18)
The definition should have $m = 1$;

$$H_0^1(\Omega) = \left\{ u \in H^1(\Omega);\ \lim_{k\to\infty} \|u - \psi_k\|_{H^1} = 0 \text{ for } \psi_k \in C^\infty_{\text{cpt}}(\Omega) \right\}.$$

p. 192: Theorem 10.13
The dimension should be n: "A function $f \in L^2(\mathbb{T}^n)$ lies in $H^m(\mathbb{T}^n)$ for ..." The 2π in the second line of the final equation should be $(2\pi)^n$.

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