EMORY MATHEMATICS DIRECTED READING PROGRAM

Program description

The Emory Math Directed Reading Program (DRP) is a graduate student-run program aiming to pair undergraduate students with graduate students to read and learn material that is not typically offered in a traditional course setting. Undergraduate students are expected to work mostly independently to read the text and attempt exercises, then meet regularly with their graduate student mentor to discuss the material.

Undergraduates must be able to meet once per week for an hour with their graduate student mentors. Meeting times will be arranged prior to the start of the semester. A prerequisite to all courses is a basic understanding of mathematical proofs, such as having taken Math 250: Foundations of Mathematics (or equivalent). Please see the course descriptions below for other prerequisite requirements.

Fall 2021 information

In Fall 2021 we are back on campus! Please see below for the courses offered (in alphabetical order) with links to their descriptions on the following page.

(1) Algebraic number theory, with Shilpi Mandal
(2) Computational algebra, with Alex Dunbar
(3) Elliptic curves, with Chris Keyes
(4) From ODEs to PDEs, with Irving Martínez
(5) Introduction to commutative algebra, with Ariella Lee
(6) Percolation theory, with Alexander Clifton
(7) Set theory, with Rohan Nair

How to apply: Interested undergraduates will fill out a short form which will be available in mid-August. The form will have options to indicate multiple courses and preferences, if desired. Check this webpage for the most up to date information.

If you have any questions or concerns, or suggestions for future DRP topics, please reach out to the program director Chris Keyes at christopher.keyes@emory.edu.

Fall 2021 course descriptions

Course name: Algebraic number theory
Instructor: Shilpi Mandal
Email: shilpi.mandal@emory.edu
Text: *TIFR pamphlets on algebraic number theory* and *Algebraic Theory of Numbers*, by Pierre Samuel.
Prerequisites: Abstract algebra I/II (Math 421,422), Abstract vector spaces (Math 321), and some commutative algebra.

Description: The idea of the course is to amalgamate one's interest in algebra with number theory. We will read through and work out the details of the TIFR pamphlets and move on to Samuel's book from there. The goal of this short course would be to build the basics necessary to concretely understand the meaning and applications of the "Lagrange's Four Squares Theorem," which states that any natural number can be represented as the sum of four integer squares. Depending upon time and interest, we'll try to go deeper into understanding the quadratic class number formula. The pacing of the course is flexible since my goal is to make learning this topic fun!

Course name: Computational algebra
Instructor: Alex Dunbar
Email: alex.dunbar@emory.edu
Text: *Ideals, Varieties, and Algorithms: an Introduction to Computational Algebraic Geometry and Commutative Algebra*, by David Cox, John Little, and Donal O'Shea.
Prerequisites: Linear algebra (Math 221 or equivalent). Some familiarity with rings (e.g. Math 421 and/or 422) and/or programming (e.g. Math 315) would be beneficial, but not necessary.

Description: One fundamental problem in algebraic geometry is to study the solution set of a system of polynomial equations in many variables. Taking inspiration from linear algebra, we want to explore the algebraic, geometric, and computational properties of such systems. In this reading course, we will start by working out the basic algebraic properties of ideals in polynomial rings and the associated geometric properties of varieties. We will then begin our study of Gröbner bases, Buchberger's algorithm, and elimination theory, the main computational tools in the course. Depending on time and and interest, the last portion of the semester can be spent exploring applications, further theory, or developing implementations of the algorithms in the course.

Course name: Elliptic curves
Instructor: Christopher Keyes
Email: christopher.keyes@emory.edu
Text: *Rational Points on Elliptic Curves*, by Joseph Silverman and John Tate.
Prerequisites: Abstract algebra I, preferably also II (Math 421, 422). Number theory (Math 328) would help, but isn't required.

Description: Elliptic curves have played a central role in number theory for over a century. While they arose from classical complex geometry, they have more recently been used in cryptographic applications, e.g. keeping online credit card transactions secure. The subject also provides a wonderful entry point to the world of arithmetic and algebraic geometry and the study of rational points on curves and abelian varieties. Some familiarity with groups and finite fields (Abstract Algebra I and possibly II) is necessary, and any additional number theoretic or analytic background will be useful.

The first goal of this course is to understand that the points on an elliptic curve have the structure of an abelian group. Indeed, this group is finitely generated, a fact known as the Mordell–Weil theorem. To understand the proof, we begin by investigating points of finite order (a.k.a. torsion points), then we will build a theory of heights to complete the proof of the theorem. We will also spend some time thinking about elliptic curves over finite fields — the setting of their cryptographic applications. This could lead to working through toy examples of elliptic curve cryptography and/or factoring algorithms, which may especcially interest those with programming experience or an interest in learning!

Course name: From ODEs to PDEs
Instructor: Irving Martínez
Email: irving.martinez@emory.edu
Text: *Partial Differential Equations*, by Evans and lecture notes from the University of Cambridge.
Prerequisites: Differential equations (Math 212) and real analysis I/II (Math 411/412). Partial differential equations (Math 351) is welcome, but not required. Exposure to measure theory is preferred, but topics requiring it can be avoided or mildly introduced.

Description: The study of differential equations began with the purpose of understanding better the dynamical systems occurring everywhere in nature. The first (ordinary) differential equations predicting the evolution of a system can be attributed to Newton and Leibniz for the description of physical laws such as gravitational forces. Then a second wave of differential equations took place with Euler, Fourier, and the creation of partial differential equations (PDEs) to explain continuum mechanics in more than one independent variable. Moreover, differential equations did not only revolutionize science but mathematics as well. For instance, their development contributed significantly to the areas of complex analysis, dynamical systems, and differential geometry. Thus, differential equations beautifully intertwines fields in pure and applied mathematics.

For this course, we will attempt to gain a deeper understanding of the qualitative analysis of ordinary differential equations (ODEs). Rather than finding techniques for "solving" equations, we will use analysis tools to better comprehend their behavior. The following topics are to be covered: existence and uniqueness theorem for ODEs, analyticity, difference between types of regularity, Arzela-Ascoli theorem, Picard-Lindelöf theorem, Cauchy-Peano theorem, Cauchy-Kovalevskaya Theorem for ODEs (with an extension to PDEs, depending on the background of the student), analysis of local and global solutions, and classification, overview, and analysis toolbox for PDEs (this can be flexible according to time allowance and background of student).

Course name: Introduction to commutative algebra
Instructor: Ariella Lee
Email: ariella.lee@emory.edu
Text: *Introduction to commutative algebra*, by Michael Atiyah and Ian Macdonald.
Prerequisites: Abstract algebra I (Math 421) or some familiarity with groups and rings would help, but isn't necessarily required.

Description: The goal of this course is to read through and work out the details in the first few chapters of Atiyah and Macdonald's *Introduction to Commutative algebra*. Each week, we will select a few theorems and exercises to write out in detail. We will focus on concrete examples and problem solving. Depending on time and interest, we can stop after the second chapter to apply what we have learned to geometric problems from Fulton's *Algebraic curves* and applications of the Nullstellensatz or continue with more commutative algebra. As this is meant to be introductory, a student of any level is welcome (though if there is no familiarity with algebra then more meetings may be necessary).

Course name: Percolation theory
Instructor: Alexander Clifton
Email: aclift2@emory.edu
Text: *Percolation*, by Béla Bollobás and Oliver Riordan.
Prerequisites: Some familiarity with probability concepts, including expected values, exposure to graph theory, and familiarity with concepts of supremum and infimum

Description: : If you submerge a porous stone in water, will its center get wet? You can model this phenomenon by thinking of the interior of the stone as a network of passages that are each either open or closed. If there is a path from the outside of the stone all the way to its center, consisting of just open passages, then the water will be able to seep in and reach the center. More generally, we can consider a grid where each pair of adjacent grid points is connected or not with some fixed probability, μ. If we have a low connection probability μ, we expect to get lots of small connected components while if we have a large μ, we expect there to be a very large (infinite if the original grid is) component. It turns out that there will be a distinct threshold value for μ at which the behavior changes and in some cases, we can even calculate this value exactly. Percolation theory has applications to electrical networks, magnetism, and even epidemics!

Course name: Set theory
Instructors: Rohan Nair
Email: rohan.nair@emory.edu
Text: *Set Theory and Logic*, by Robert R. Stoll.
Prerequisites: Foundations of math (Math 250) or similar exposure to proofs.

Description: The theory of sets is home to some of the most philosophically challenging and counter-intuitive results in all of mathematics. The goal of this course is to explore set theory at a level deeper than is covered in Foundations of Mathematics, and our initial plan is to work through as much of Chapters 2, 3, 5, and 7 of Stoll's *Set Theory and Logic* as possible. Depending on interest and time, however, we might branch out into other readings and topics.