Math 421 Problem Set
September 6, 2022

1. Recall from class that D_{2n} is the dihedral group, with presentation

$$D_{2n} = \langle r, s \mid r^n = s^2 = 1, rs = sr^{-1} \rangle$$

   (a) If $n = 2k$ is even and $n \geq 4$, show that r^k is the only nonidentity element of D_{2n} that commutes with all elements of D_{2n}.

   (b) If n is odd and $n \geq 3$, show that the identity is the only element that commutes with all elements of D_{2n}.

   (c) For $n = 1$ and 2, show that D_{2n} is abelian. To which familiar groups are these isomorphic?

2. Give presentations for the groups $\mathbb{Z}/4\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.