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Multilevel Optimization methods for Mutual Information registrations

by

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Computational methods for Mutual Information registration

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Abstract

Working note, not for general distribution

1 Numerical Examples

In this section we present two numerical examples illustrating the performance of our scheme. Note that we are not intend to discuss the results from a medical point of view. Also, we do not intend to discuss whether mutual information is an appropriate distance measure or not.

For both examples we used a multilevel approach, where on each level we start with an u^0 obtained by prolongating the solution of the coarser level ($u^0 = 0$ on the coarsest level), perform an non-regularized linear pre-registration followed by a non-linear registration, where only the non-linear parts are regularized; see [5] for details. As a regularizer we used the elastic potential and a regularization parameter $\alpha = 10^{-2}$. The linear and non-linear iterations are stopped, if the change in u is smaller than 10^{-5} or if the number of iterations exceeds 50. For the density estimation, we used one global and fixed $\sigma = 32$ and a 32×32 discretization of the grayvalue space of R and T.

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We used five levels, such that the image dimensions are $2^k \times 2^k$, $k = 4, \ldots, 8$. Intermediate results for both examples are displayed in Figure 1 and Figure 2. For three representative levels, we show our density approximation, the registered template $T^{(\ell)}(u^*)$ and the deformed grid overlayed to the unregistered template $T^{(\ell)}(0)$.

As expected and in accordance to the theory, most of the correction has been performed on the coarser level, such that on the finer levels we are left with one correction step, see Table 1 for a detailed summary of the results. Here, the improvement factors are defined by

$$
\begin{aligned}
i_{\text{coarse}} &= \mathrm{MI}(R^{(\ell)}, T^{(\ell)}(u^{\text{coarse}}))/\mathrm{MI}(R^{(\ell)}, T^{(\ell)}(0)), && \text{(1a)}\\
i_{\text{linear}} &= \mathrm{MI}(R^{(\ell)}, T^{(\ell)}(u^{\text{linar}}))/\mathrm{MI}(R^{(\ell)}, T^{(\ell)}(0)), && \text{(1b)}\\
i_{\text{elastic}} &= \mathrm{MI}(R^{(\ell)}, T^{(\ell)}(u^{*}))/\mathrm{MI}(R^{(\ell)}, T^{(\ell)}(0)), && \text{(1c)}
\end{aligned}
$$

where u^{coarse} is the starting guess obtained from the coarser level ($u^{\text{coarse}} = 0$ on the coarsest level), u^{linar} is the numerical solution of the affine linear registration, and u^* is the solution after the complete registration on that level.

1.1 X-ray of hands

Figure 1 displays registration results for two modified X-rays of human hands (a,b) and the registered template (c) in a 256×256 resolution; see also [1], [6]. This example is very interesting and challenging from a computational point of view. Firstly, the wanted transformation has large linear and also non-linear parts. Secondly, there are many local minima. For example, considering only a rotation, a local minima has to be expect whenever a finger of the transformed template hand meets a finger of the reference hand. ???. Moreover, for this example, the results are easy to understand and interpreted.

1.2 MRI scans of a brains

Our second example focus on registration of multi-modal images. As a reference we took a real T1-weighted magnetic resonance image (MRI) of the brain of a periventricular leukomalacia patient, showing some abnormally enlarged ventricles; cf. Figure 2(a). As a template we took an idealized but heavily deformed T1-weighted scan from the BrainWeb simulator [2]; see also [3] and [4]. Note the different white matter pattern in the two brains.

Table 1: Results for the hand and brain example with respect to different levels. On level ℓ, images are $n \times n$, #it denotes the number of iteration spend on this level (we used a maximum of 50 for the linear and the non-linear parts), time is the cputime in seconds, i_* denotes the improvement rates (cf. (eq:reduction 1)). tab:Hands-MRI

X-rays of hands								
level	n	linear		elastic		improvement		
		#it	time	#it	time	i_{coarse}	i_{linear}	i_{elastic}
1	16	46	8.8	50	9.3	1.0000	1.0935	1.1133
2	32	35	22.9	50	34.6	1.1070	1.1124	1.1339
3	64	1	2.4	1	2.8	1.1253	1.1252	1.1252
4	128	1	10.3	1	12.1	1.1160	1.1160	1.1160
5	256	1	49.5	1	71.3	1.1087	1.1087	1.1087

MRIs of brains								
level	n	linear		elastic		improvement		
		#it	time	#it	time	i_{coarse}	i_{linear}	i_{elastic}
1	16	41	7.7	50	9.3	1.0000	1.0816	1.1154
2	32	7	4.5	50	34.3	1.0997	1.1000	1.1312
3	64	1	2.4	1	2.7	1.1165	1.1165	1.1165
4	128	1	9.7	1	12.4	1.1039	1.1039	1.1039
5	256	1	47.2	1	66.8	1.0953	1.0953	1.0953

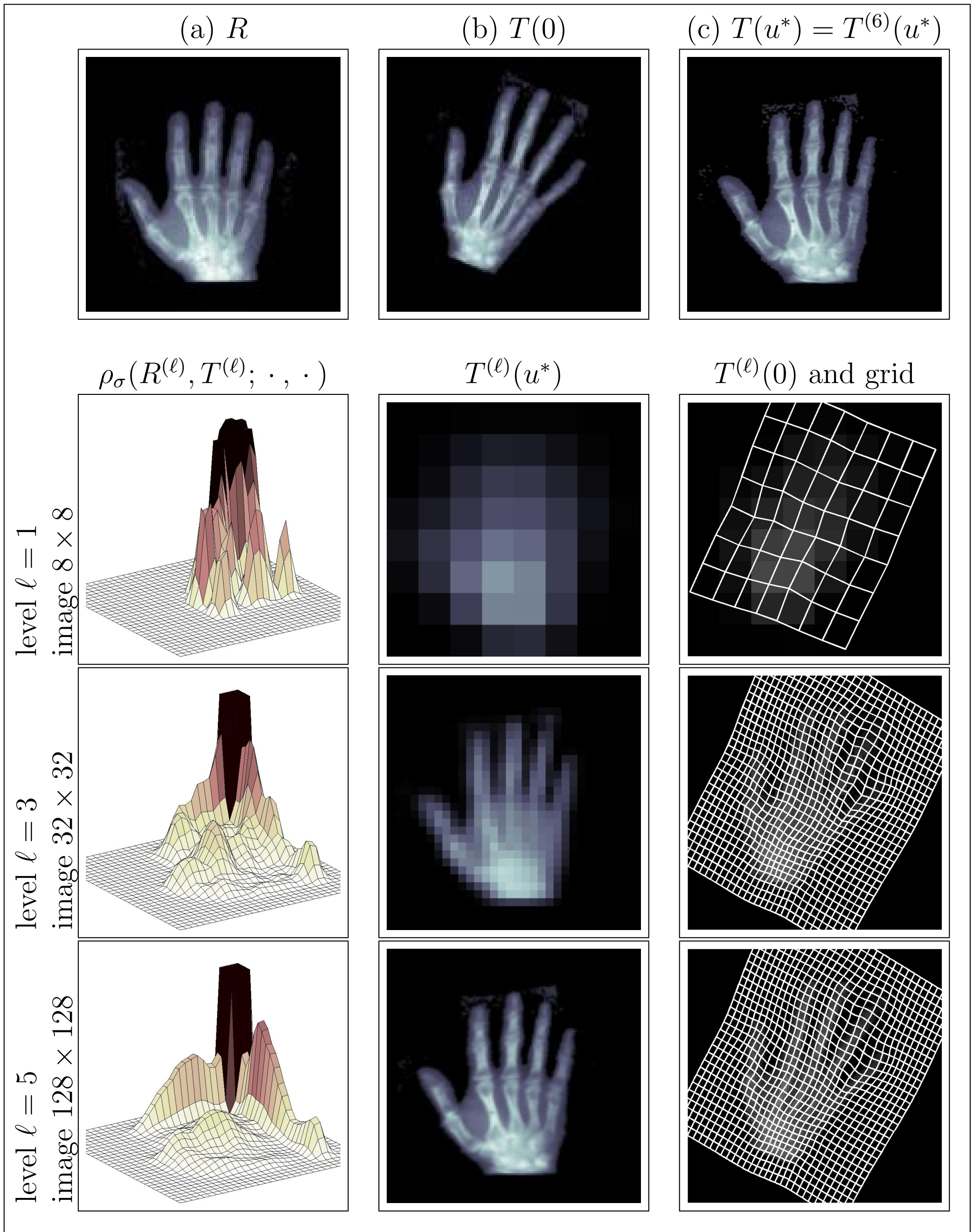

Figure 1: Registration results for two modified X-rays of human hands. fig:Amits-hands

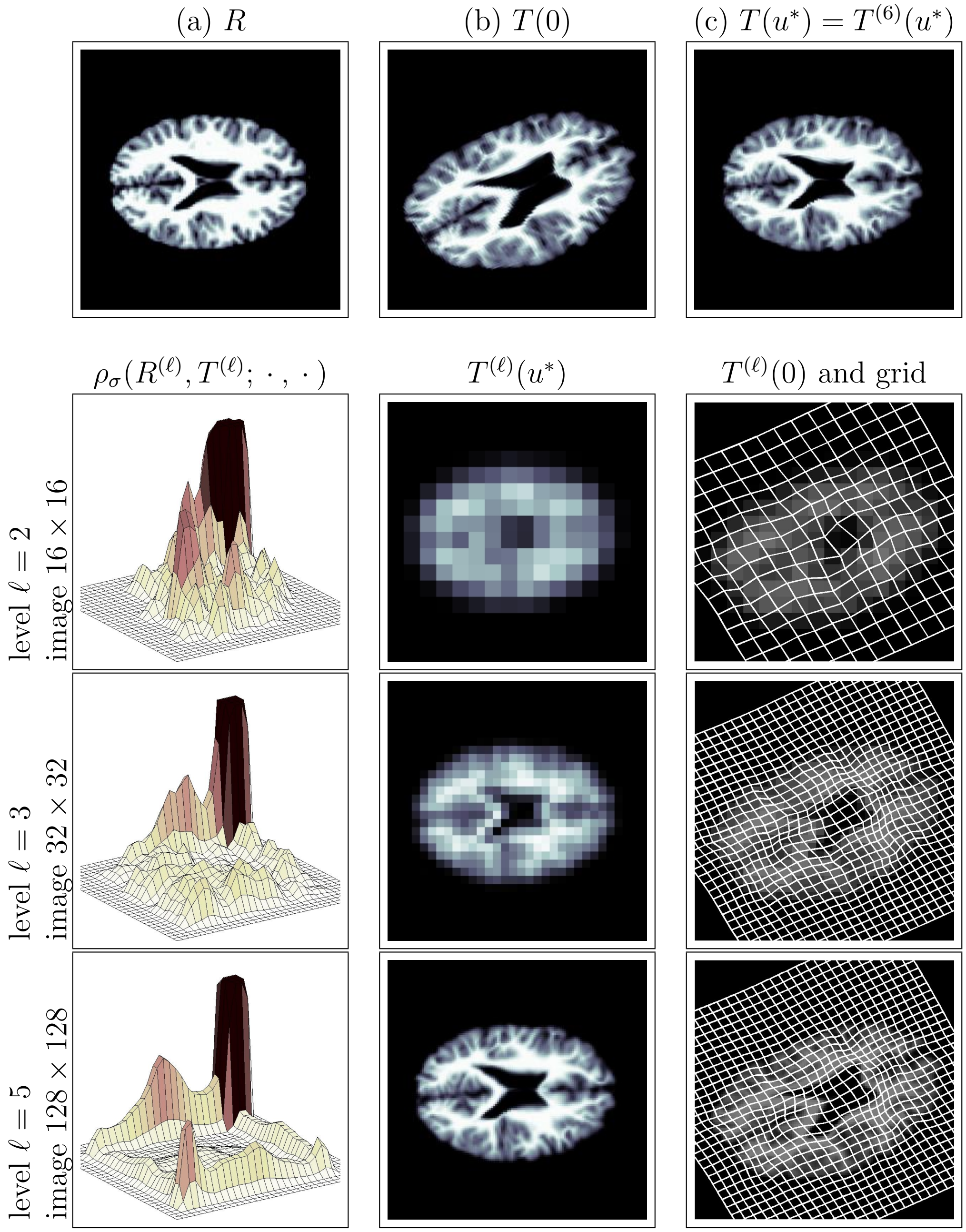

Figure 2: Two modified X-rays of human hands. fig:MRI

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