Data Assimilation for Glacier Modeling

Corcoran Emily[1], Knudsen Logan[2], Park-Kaufmann Hannah [3]

[1]Department of Mathematical Sciences, College of Science and Liberal Arts, New Jersey Institute of Technology
[2]Department of Mathematics, College of Science, Texas A&M University
[3]Department of Mathematics, Division of Science, Mathematics, and Computing, Bard College

Motivation

Research has shown that climate change will likely impact storm surge inundation and make modeling this process more difficult [Camelo et al, 2020]. Sea-level rise caused by climate change plays a part in this impact. To better model sea-level rise, glaciers can be modeled. Marine-Terminating Glaciers have a natural flow towards the ocean, which contributes to sea level rise [Robel et al., 2019]. By the year 2300, the Antarctic ice sheet is projected to cause up to 3 meters of sea level rise globally [Robel, 2015].

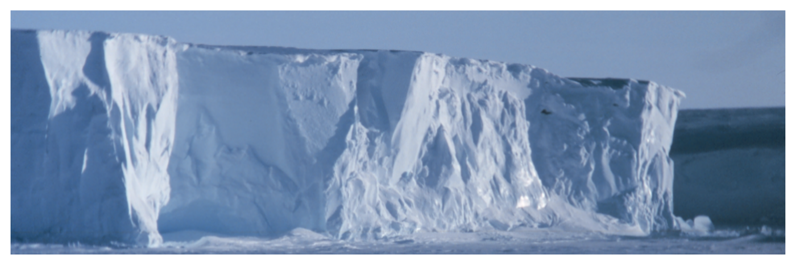

Fig. 1: Michael Van Woert, National Oceanic and Atmospheric Association (NOAA) NESDIS, ORA

Due to the severe impacts of glacial melting, modeling changes in ice sheets is an important task. There are challenges to modeling sea level rise, as ice sheet instability leads to significant sea-level rise uncertainty [Robel et al., 2019].

The Model

The ice sheet model described here aims to describe the changes in ice mass of marine-terminating glaciers, which may be impacted over time by climate change [Robel et al., 2018].

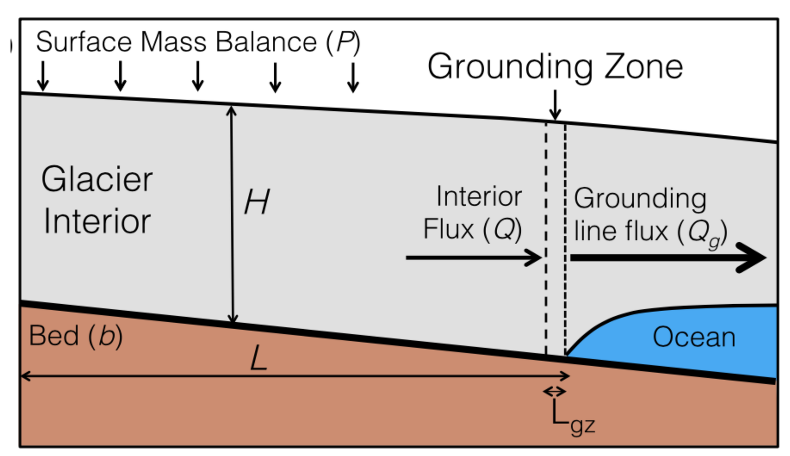

Fig. 2: Diagram of a marine-terminating glacier

A glacier can be represented with a simplified box model that has a length L, precipitation P, and height and flux at the grounding line h_g and Q_g. The two-stage model used here incorporates a nested box into the system. This new box has a thickness, H, and an interior flux, Q. The change in length and height of the glacier can be described with these differential equations:

$$\frac{dH}{dt} = P - \frac{Q_g}{L} - \frac{H}{h_g L}(Q - Q_g)$$

$$\frac{dL}{dt} = \frac{1}{h_g}(Q - Q_g)$$

The model used here utilizes the Ensemble Kalman Filter, a data assimilation technique that uses observations and the covariance matrices to adjust the model's output to be closer to reality.

Methods

1. Sensitivity Analysis

Sensitivity analysis studies how various sources of uncertainty in a mathematical model contribute to the model's overall uncertainty. We vary each parameter by +-10 percent of the nominal values originally given in our model code, and if the outputs vary significantly, then the output is sensitive to the specification of the input distributions.

2. Data Assimilation

Data assimilation is a method to move models closer to reality using real world observations by readjusting the model state at specified times.

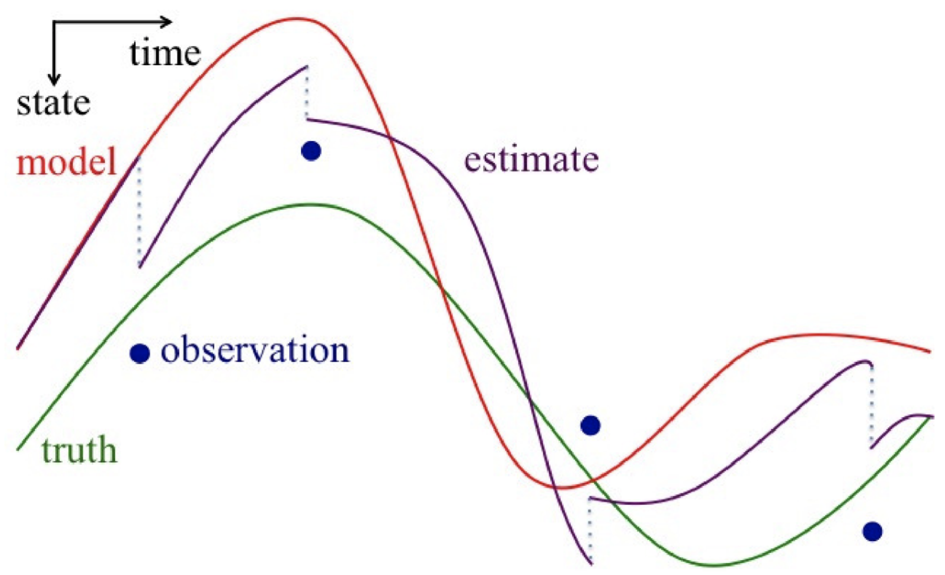

Fig. 3: Data Assimilation

2.1 Ensemble Kalman Filtering

The EnKF is a nonlinear version of the Kalman filter fit for large problems. The covariance matrix is replaced by the sample covariance.

Forecast:

$$\mathbf{x}^f_{k,i} = \mathcal{M}_{k,i}\mathbf{x}^a_{k-1,i} + w_{k,i} \tag{1}$$

$$\mathbf{P}^f_{k,i} = \frac{1}{N-1}\sum_{j=1}^{N}(\mathbf{x}^f_{k,i} - \mathbf{x}^f_k)(\mathbf{x}^f_{k,i} - \mathbf{x}^f_k)^T \tag{2}$$

Analysis:

$$\mathbf{x}^a_{k,i} = \mathbf{x}^f_{k,i} + \mathbf{G}_{k,i}(\mathbf{y}_{k,i} + \mathbf{v}_{k,i} - \mathcal{H}_{k,i}\mathbf{x}f_{k,i}) \tag{3}$$

$$\mathbf{P}^a_{k,i} = (\mathbf{I} - \mathbf{G}_{k,i}\mathbf{H}_{k,i})\mathbf{P}^f_{k,i} \tag{4}$$

$$\mathbf{G}_{k,i} = \mathbf{P}^f_{k,i}\mathbf{H}^T_{k,i}(\mathbf{H}\mathbf{P}^f_{k,i}\mathbf{H}^T_{k,i} + \mathbf{R}_{k,i})^{-1} \tag{5}$$

Here, in the forecast: (1) State forecasted from the prior probability distribution(PD), (2) Error covariance, and in the analysis: (3) State from data posterior PD, where (4) New prediction, and (5) Kalman gain.

Sub-/superscripts: f = forecast, a = analysis, k = time, and i indexes the ensembles.

Results

3. Model Parameter Sensitivity

Varying the sill parameters by +/-10% caused a greater degree of variation in the length and height of the glacier than the initial conditions and precipitation values.

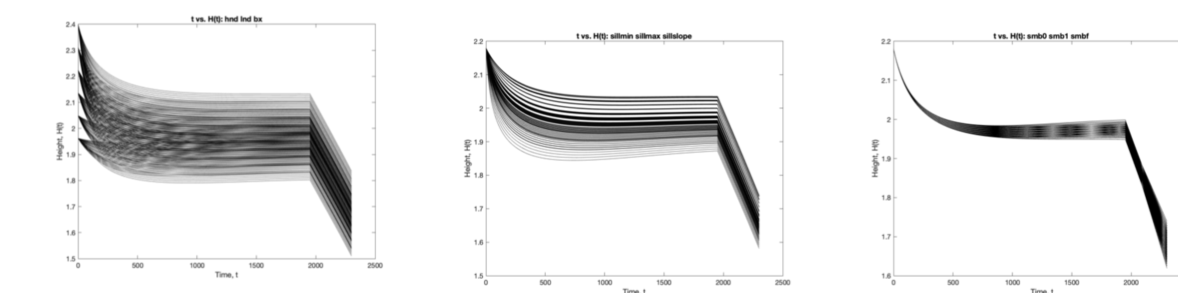

Figure: ±10% initial conditions
Figure: ±10% sill
Figure: ±10% SMB

Fig. 4: Sensitivity analysis graphs for height of glacier over time with initial conditions (left), sill parameters (middle) and SMB parameters (right) varied

4. Ensemble Size

The mean squared difference hovers around a consistent value starting at about 7-10 ensembles.

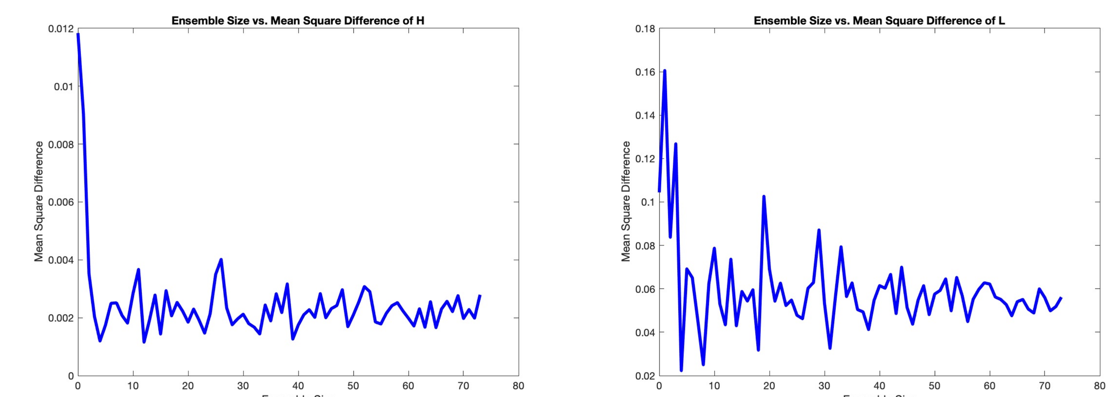

Fig. 5: Graphs of mean squared difference of the analysis from the truth after the last observation in 2020.

5. Assimilation Frequency

We grouped the data assimilation schemes into two categories: observations made only before 1900 and observations made only after 1950. It was found that making observations every 19 years before 1900 or yearly after 1950 allowed the truth to be largely recovered.

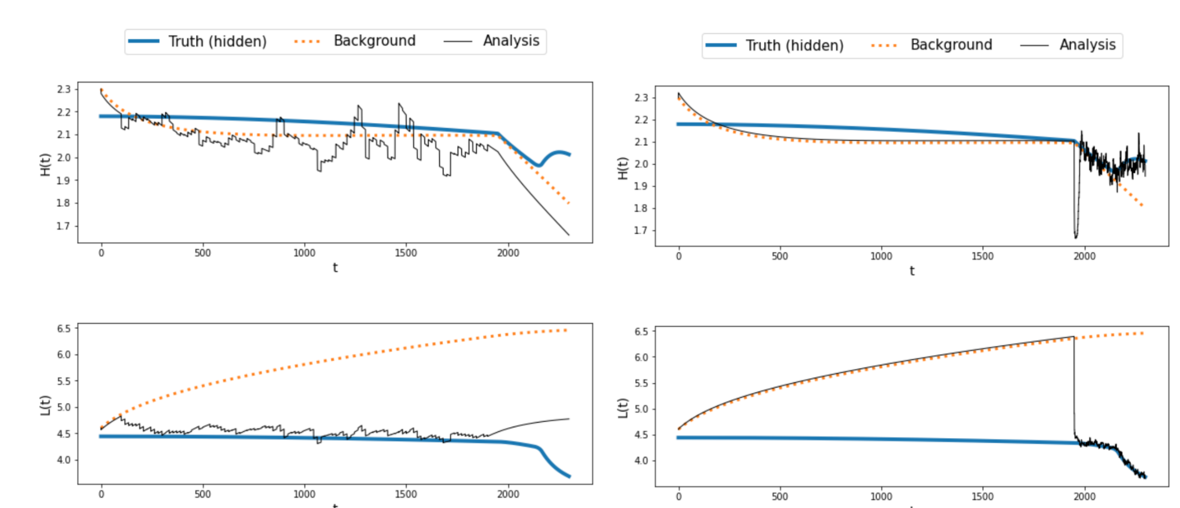

Fig. 6: Left: Data assimilated prior to 1900 every 19 years
Right: Data assimilated from 1950 to 2300 once per year

Discussion

- We have found the EnKF improves model runs initialized off of incorrect initial conditions or parameters, providing us with better models of future glacier melt.
- Our research determines the necessary number of observations needed to get an accurate model run.

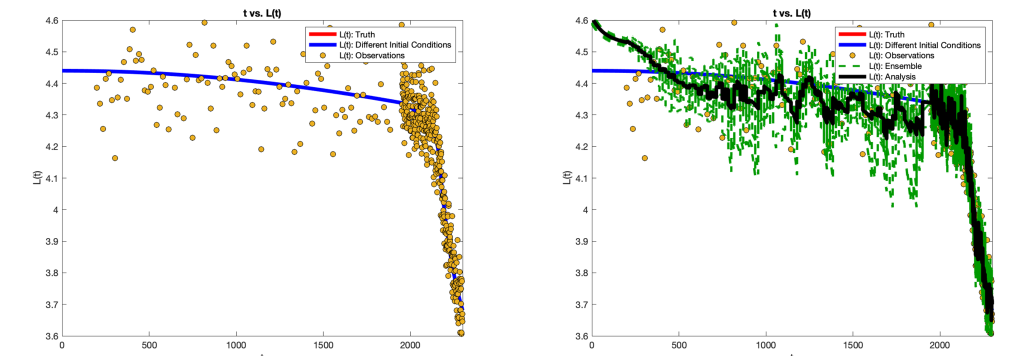

Fig. 7: Plot of the true state of L(t) with observations (left) and L(t) with ensemble and final analysis (right).

Future Directions

As glacier flux contributes significantly to sea-level rise, ice sheet models can be very helpful for modeling other geophysical processes, such as hurricane storm surge and sea-level rise inundation.

Acknowledgements

This work is supported in part by the US National Science Foundation awards DMS-2051019 and DMS-1751636. We would like to thank Dr. Talea Mayo for her mentorship throughout this project and Dr. Alexander Robel for his guidance and ice sheet model.

References

Camelo, J., Mayo, T.L., Gutmann, E.D., 2020. Projected climate change impacts on hurricane storm surge inundation in the coastal united states. Frontiers in Built Environment.

Robel, A., 2015. The long future of Antarctic melting. Nature 526, 327–328.

Robel, A., Seroussi, H., Roe, G.H., 2019. Marine ice sheet instability amplifies and skews uncertainty in projections of future sea-level rise. PNAS 116, 14887–14892. URL: https://doi.org/10.1073/pnas.1904822116.

Robel, A.A., Roe, G.H., Haseloff, M., 2018. Response of marine-terminating glaciers to forcing: time scales, sensitivities, instabilities, and stochastic dynamics. Journal of Geophysical Research: Earth Surface 123, 2205–2227. URL: https://doi.org/10.1029/2018JF004709

Fig. 8: Scan this for more information about our project