# Imaging & Mathematics

Today, we will

- Determine the difference between infinitely number of solutions and a unique solutions as it applies to x-ray imaging.
- Solve a system of equations using Gaussian Elimination

Agenda

1. Warm-UP -   Problem of the Day!
2. Notice / Wonder - Can Mathematics be used to save people's lives?
3. Guided Practice - Imaging and X-rays!
4. Working Together - KWL & My Square!
5. We DO -   Example on Infinitely Number of Solutions!
6. You DO -   Learning Practice, One Unique Solution Problem!
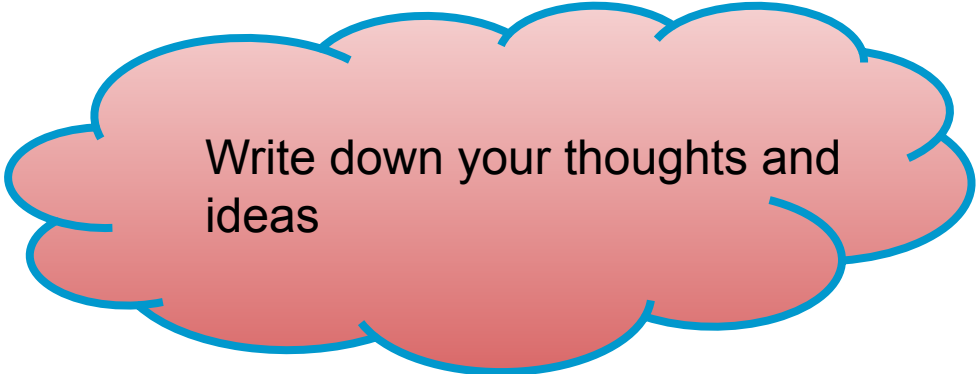7. Summary -   Difference between infinitely number and unique solutions!

Problem of the Day!

Solve the following Kakuro puzzle.
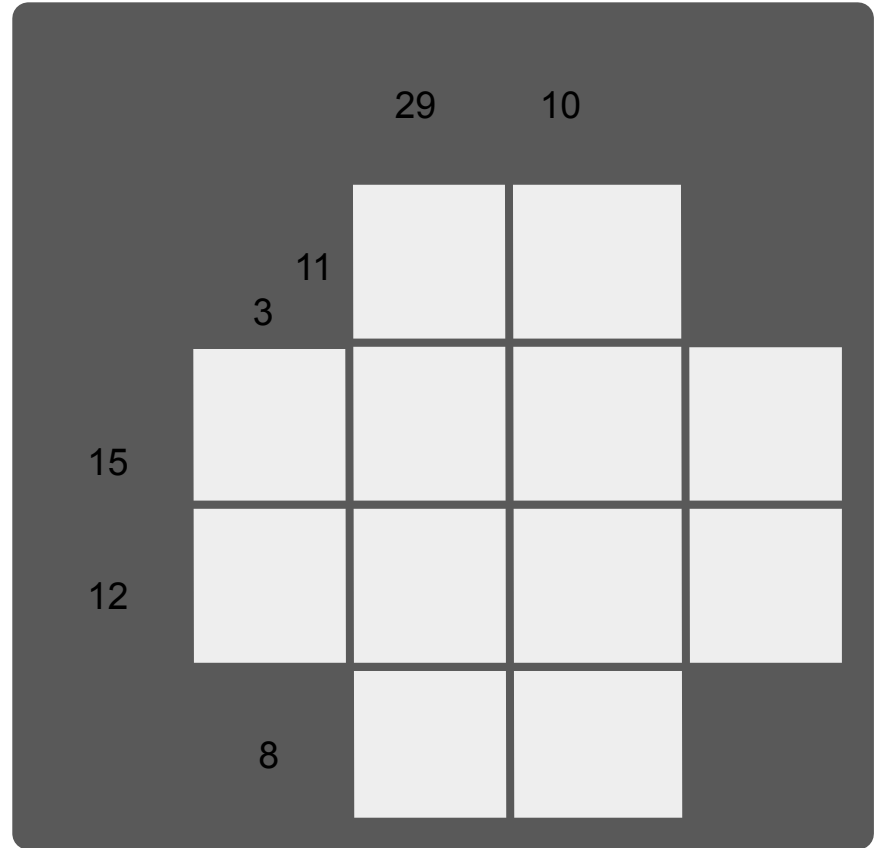
Can _**mathematics**_ be used to save people's lives?

How do you think mathematics can save people's lives?

Write down your thoughts and ideas

SHARE OUT

KWL - Know, Wonder, Learned

How Math Can Save Your Life: Tomography
Why Do Math? Article

Know	Wonder	Learned

Round Robin: Take turns in your group and talk about what your paragraphs are about.

X-Rays & Mathematics

Learning about X-Ray Imaging.
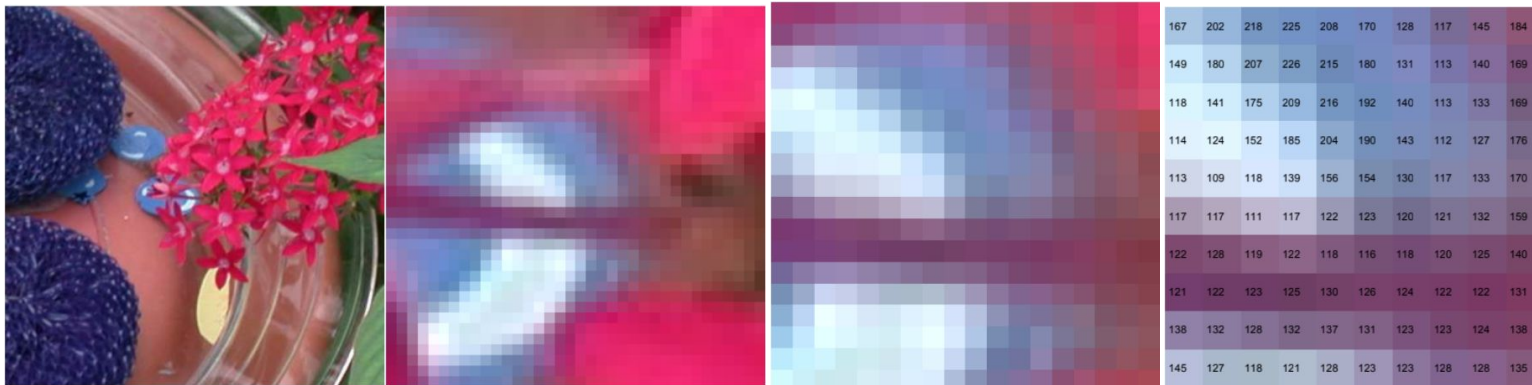
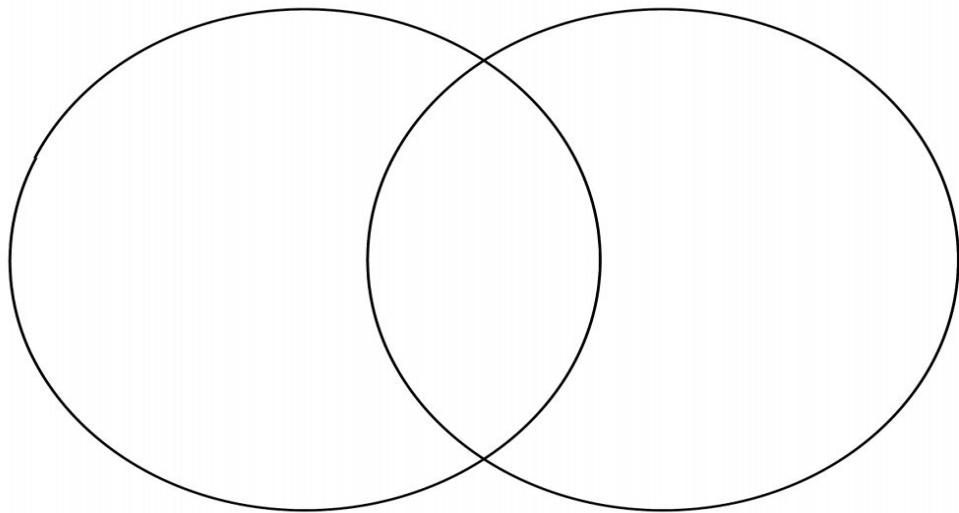# X-Rays & Mathematics

Learning about X-Ray Imaging.

Teacher will play video on X-Ray Imaging.

Each puzzle has a solution that is unique.

Teacher may give the definition for a Well-Posed problem here.

Why do imaging problems tend to be ill-posed?

1. The sources and detectors may not be perfectly calibrated; they may record noisy measurements.

2. Patients may move.
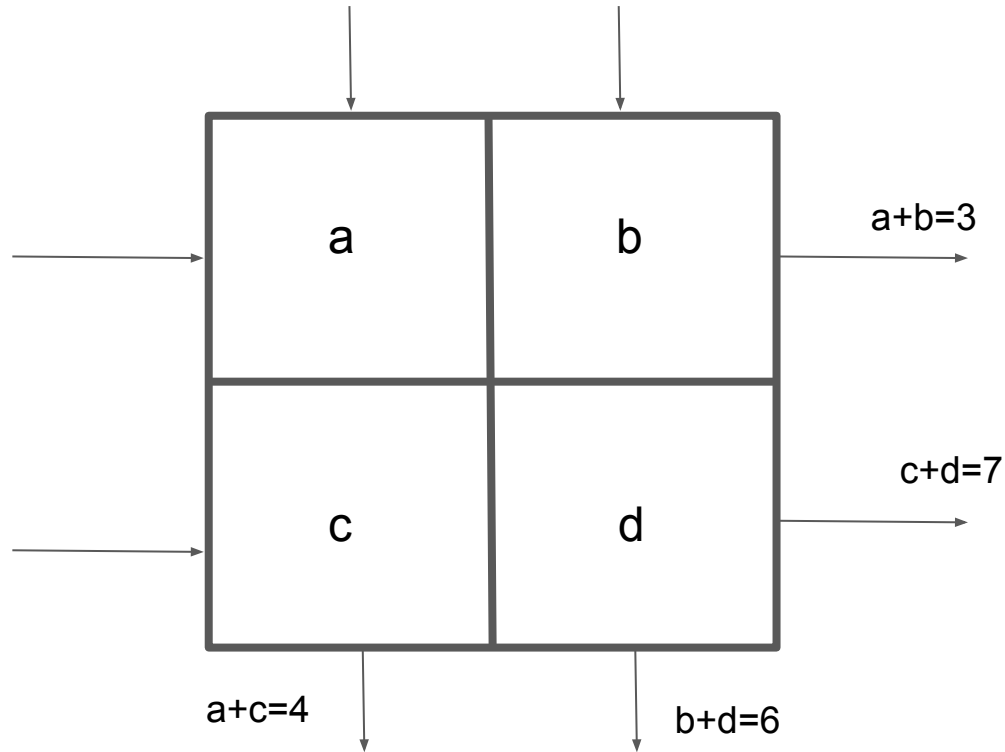
PARTNER TASK

What goes in my Square?

Directions:

1) Fill in the square with digits between 1 and 9.
2) Sum the columns and rows of your square.
3) Tell your partner the sums of the columns and rows.
4) Have your partner guess what numbers goes in the square.

I discovered that …

GUIDED PRACTICE

Finding an infinite number of solutions.

a	b
c	d

a+b=3

c+d=7

a+c=4

b+d=6

1. Write down equations like this:

$$a + b \quad\quad = 3$$
$$c + d = 7$$
$$a + \quad c \quad = 4$$
$$b \quad\quad + d = 6$$
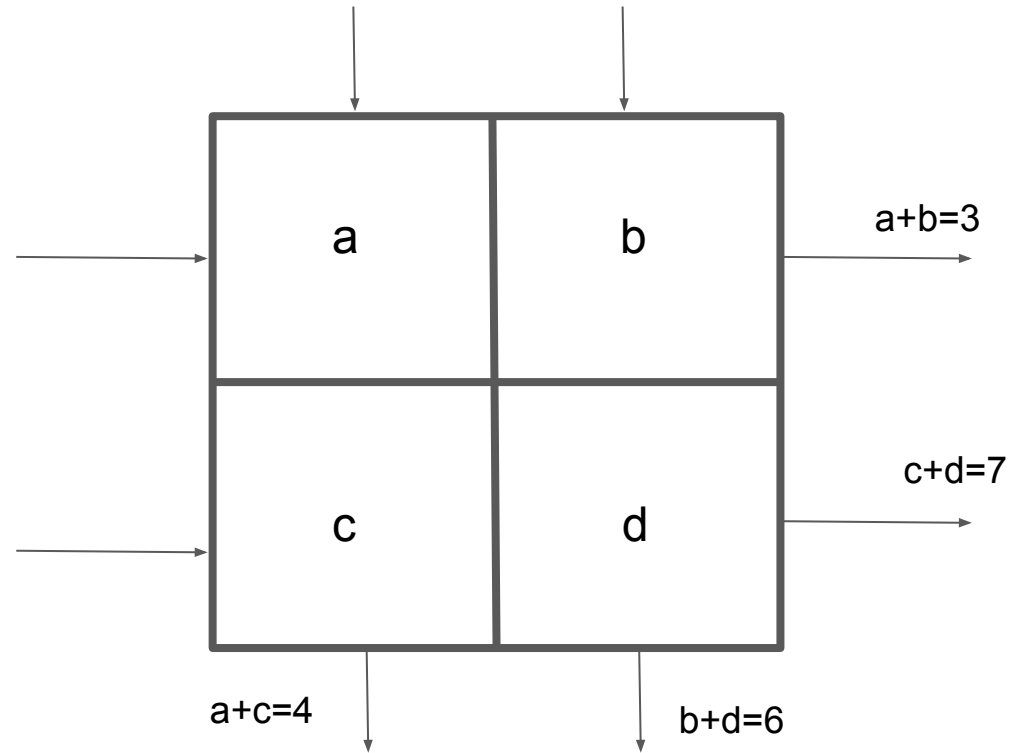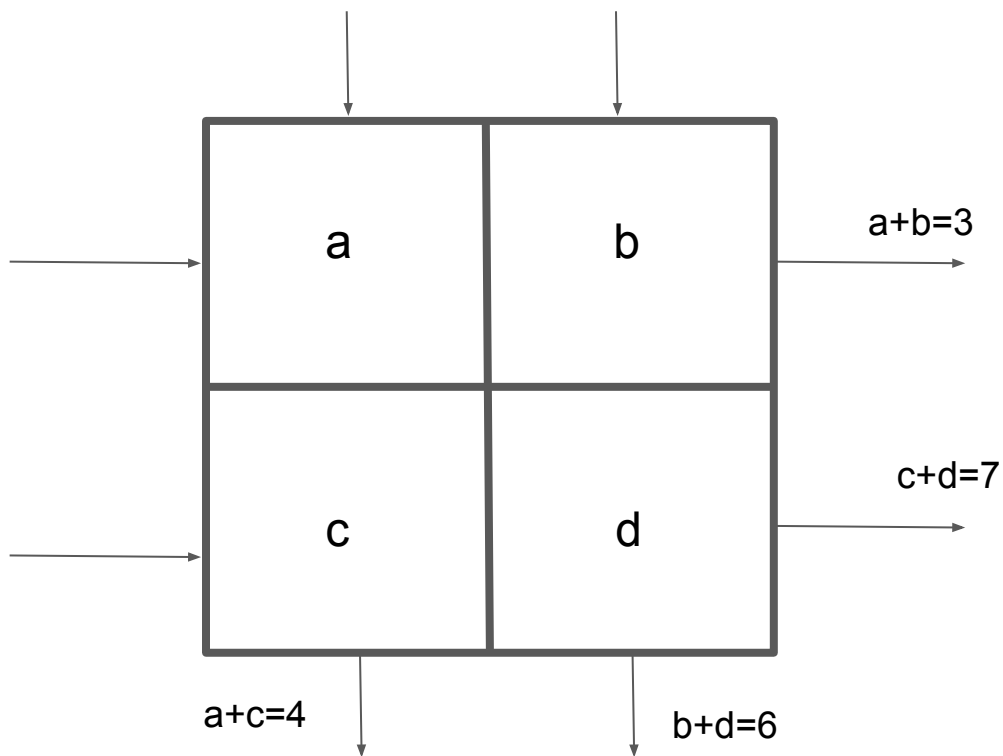
2. Write down the matrix, in the form of Ax=b.

$$
\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 4 \\ 6 \end{pmatrix}
$$

x- true image solution   b- detector data

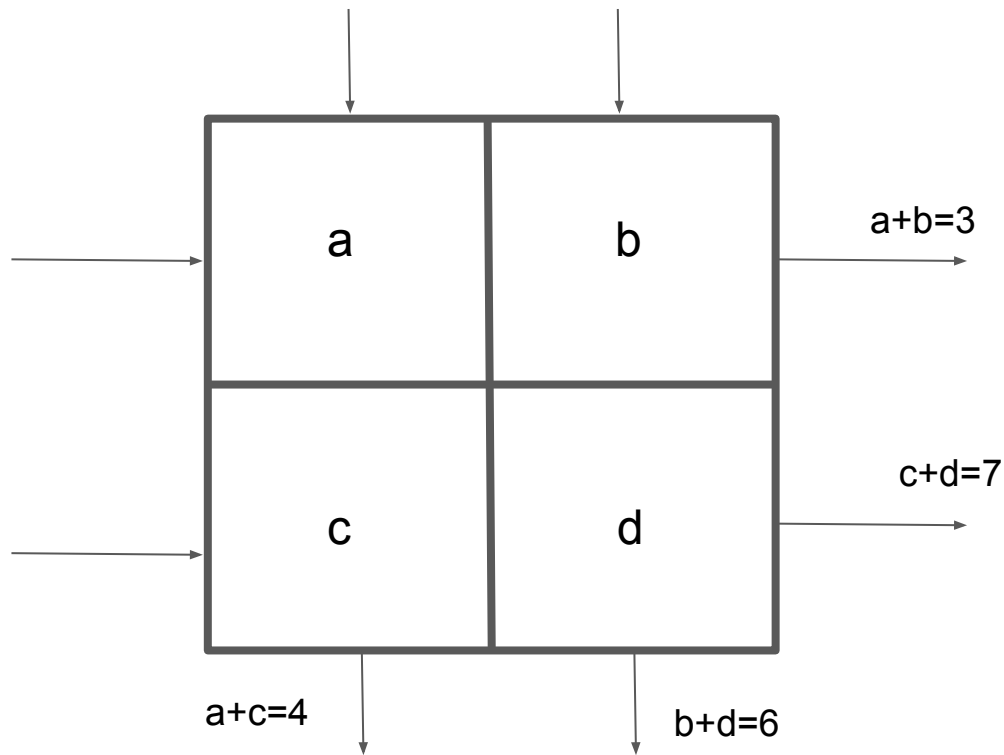$$
x = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \qquad b = \begin{pmatrix} 3 \\ 7 \\ 4 \\ 6 \end{pmatrix}
$$

a	b	a+b=3
c	d	c+d=7

a+c=4   b+d=6

3. Write down an augmented matrix:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & | & 3 \\ 0 & 0 & 1 & 1 & | & 7 \\ 1 & 0 & 1 & 0 & | & 4 \\ 0 & 1 & 0 & 1 & | & 6 \end{pmatrix}$$

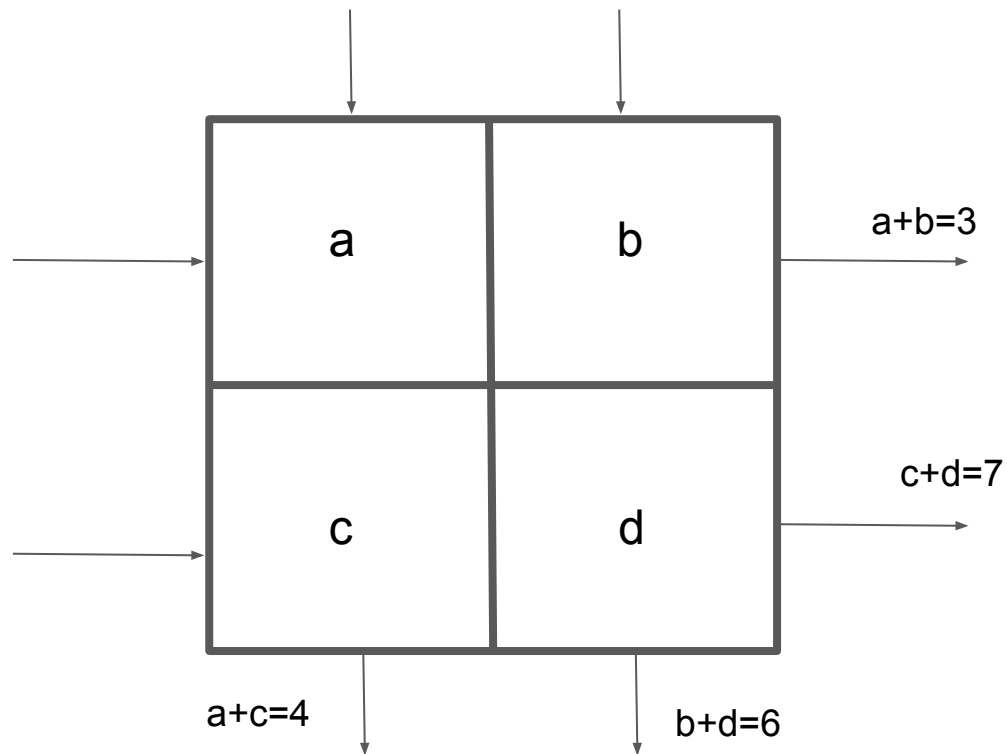| a	b
c	d

a+b=3

c+d=7

a+c=4

b+d=6

4. Solve by using Gaussian Elimination.

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 7 \\ 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 & 6 \end{array}\right)$$
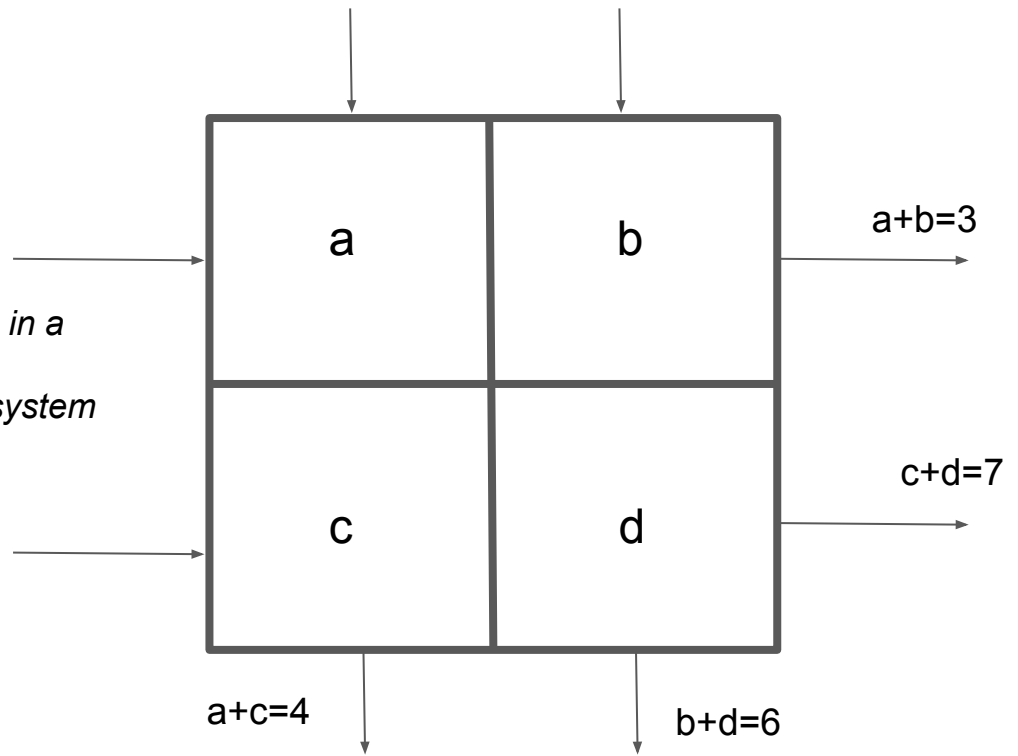
$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$
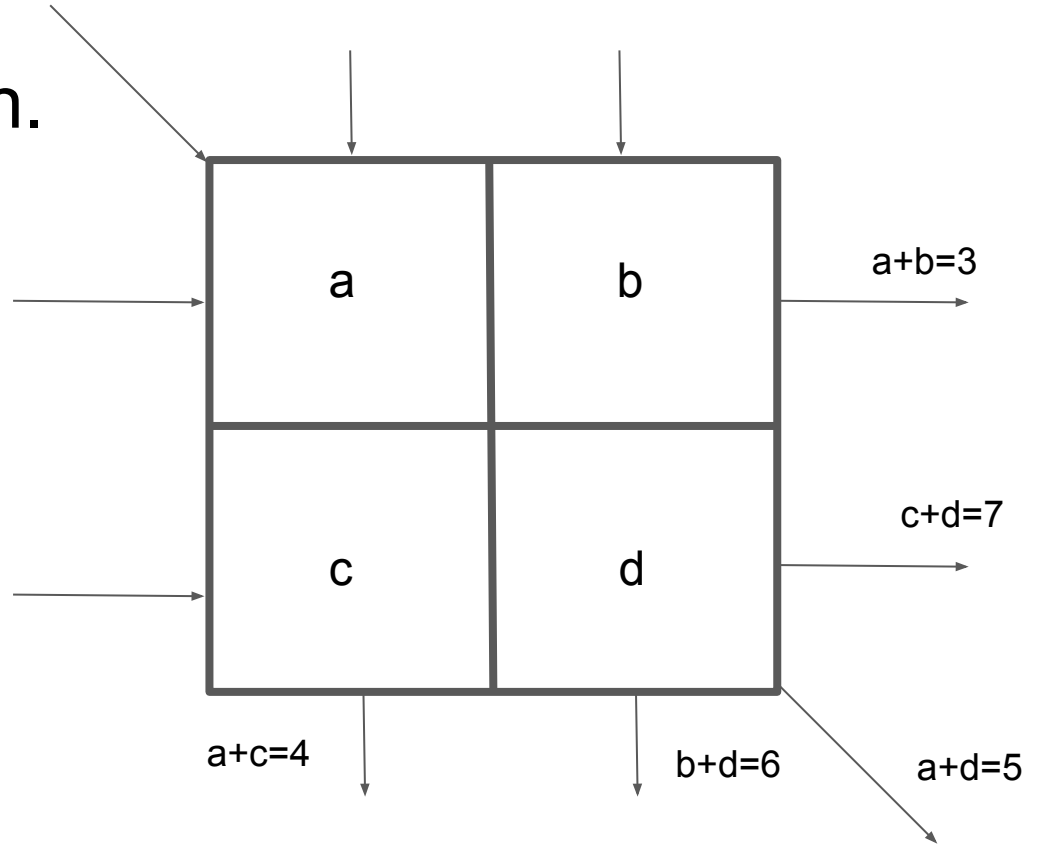
a+b=3

c+d=7

a+c=4

b+d=6

| a | b |
| c | d |

1. *What does it mean to have 0=0 in a system of equations?*
2. *How many solutions does this system equations have?*
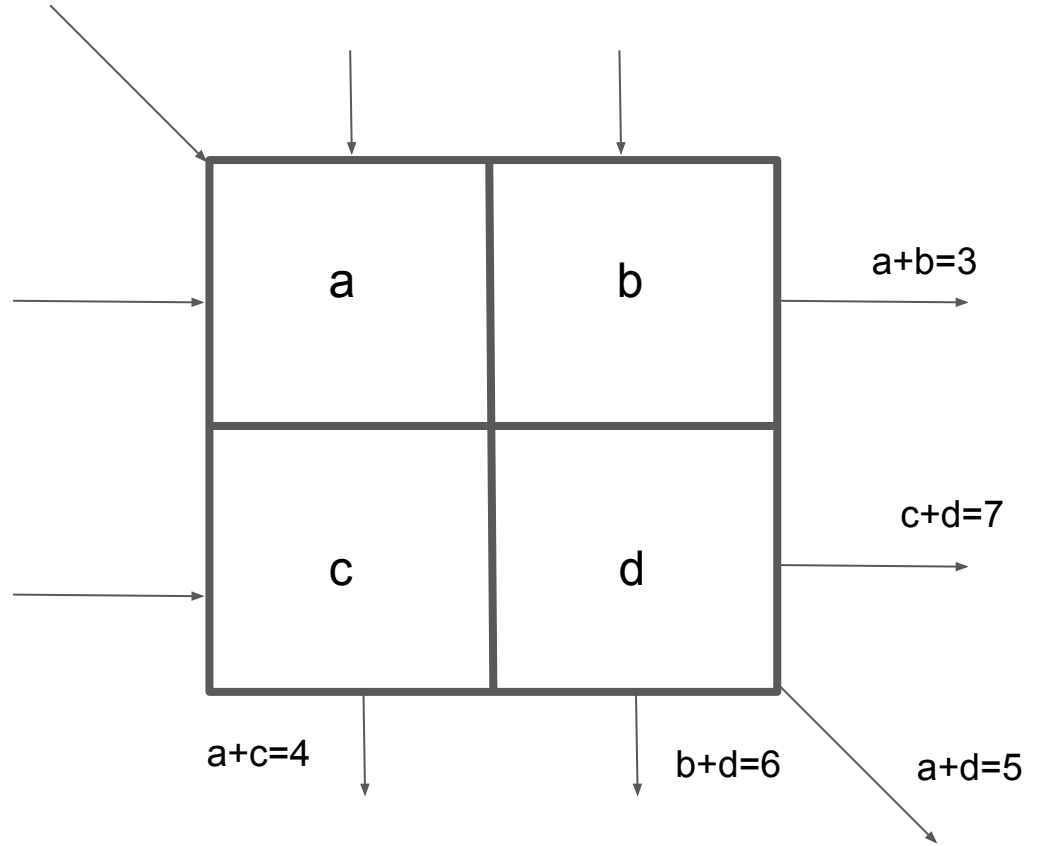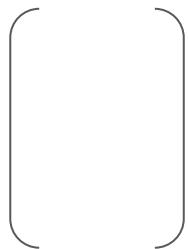
5. Write down the solution.

$a+b=3$

$c+d=7$

a	b
c	d

$a+c=4$

$b+d=6$

PRACTICE

Finding a unique solution.

a	b
c	d

a+b=3

c+d=7

a+c=4

b+d=6

a+d=5

1. Write down the equations.

a | b   a+b=3

c | d   c+d=7

a+c=4   b+d=6   a+d=5

2. Write down the matrix, in the form of Ax=b.

$$\begin{pmatrix} & \\ & \\ & \end{pmatrix}$$

x- true image solution   b- detector data

x= $\begin{pmatrix} \\ \end{pmatrix}$ b= $\begin{pmatrix} \\ \end{pmatrix}$

a	b	a+b=3
c	d	c+d=7

a+c=4   b+d=6   a+d=5

3. Write down an augmented matrix:

$$\begin{pmatrix} & \Big| & \\ & \Big| & \end{pmatrix}$$

| | a | b | a+b=3 |
| | c | d | c+d=7 |

a+c=4   b+d=6   a+d=5

4. Solve by using Gaussian Elimination.

a+b=3

c+d=7

a+c=4

b+d=6

a+d=5
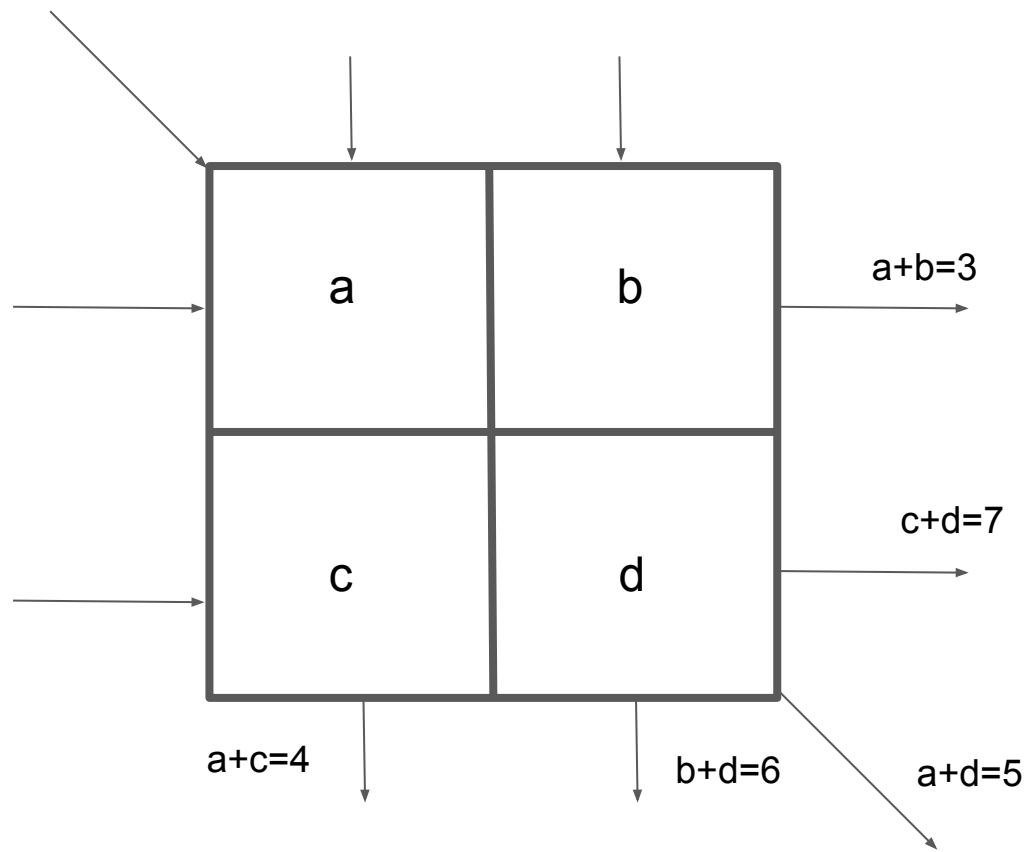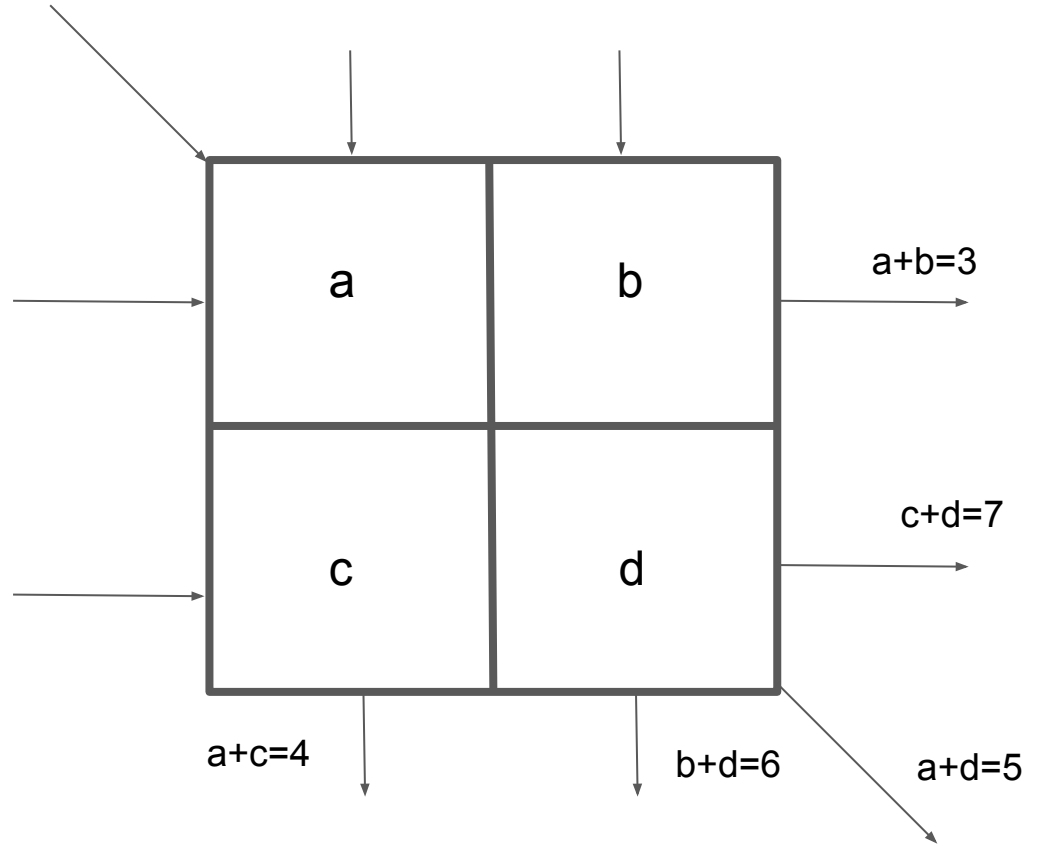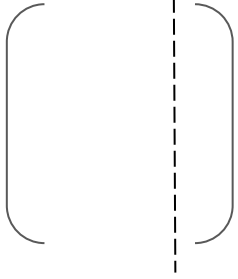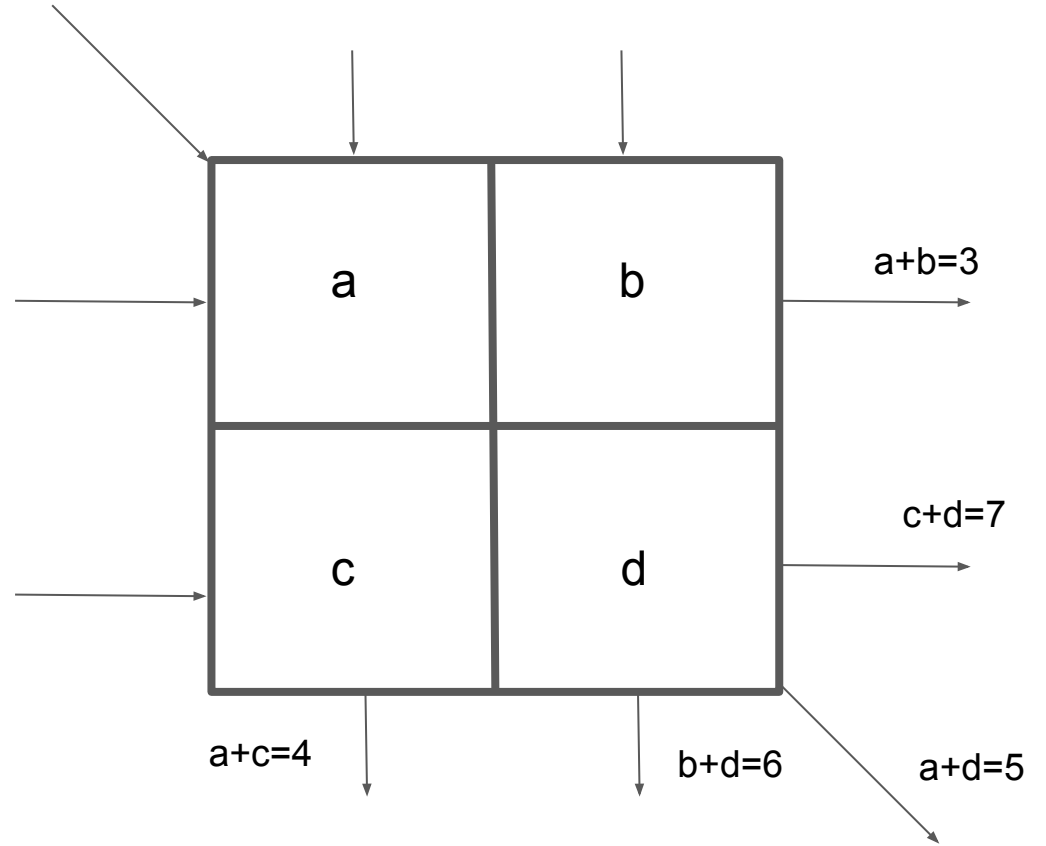
5. Write down your solution.

$$x = \begin{bmatrix} & \\ & \end{bmatrix}$$
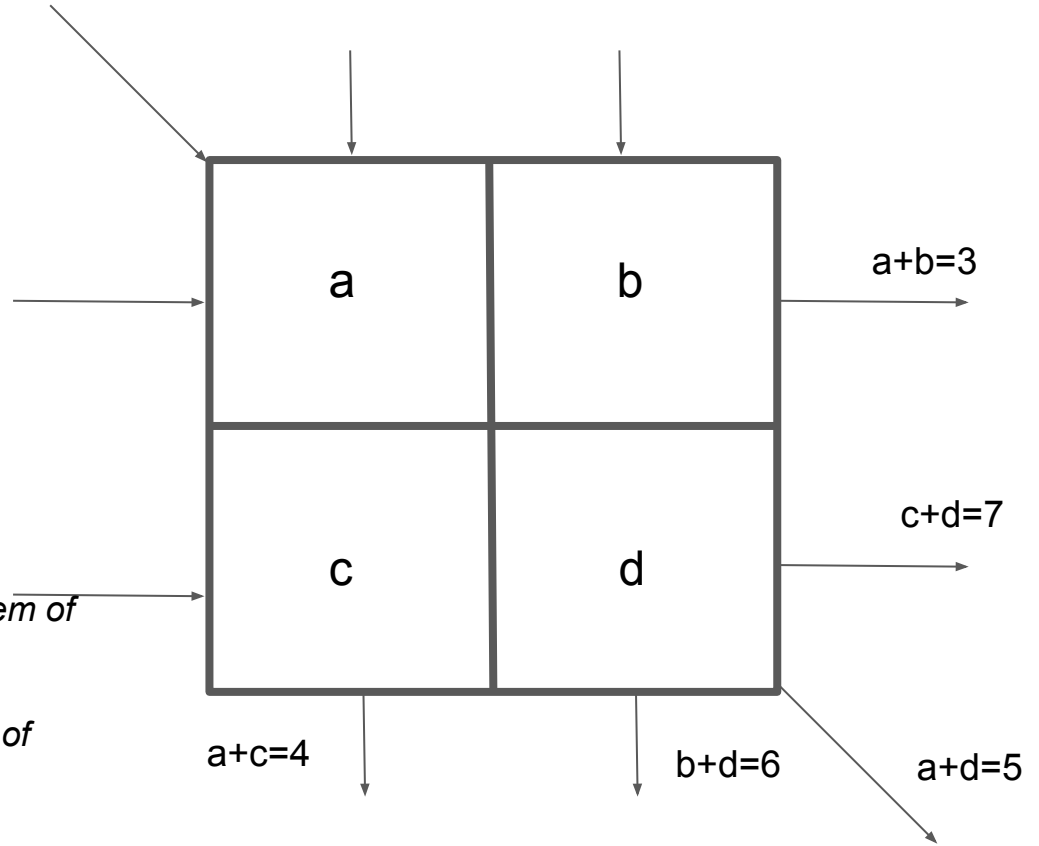
1. *Did you get a solution for this system of equations?*
2. *What kind of solution did you get?*
3. *What does it mean to get this kind of solution?*

?

a	b
c	d

a+b=3

c+d=7

a+c=4

b+d=6

a+d=5

Summary

1) *What is the difference between having an infinite number of solutions and one unique solution?*

References

❖ Avineri, Tamar, Berkely Emily, Miller, Ashley (2020). X-Ray Imaging, Mathematics, and Puzzles. Retrieved July 21, 2021, from https://asaibab.math.ncsu.edu/Module3XrayImaging.pdf
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❖ Nagy, James. Mathematics and Imaging Emory Math Circle. Retrieved June 25, 2021.