Some Problems at the Intersection of Algebra, Geometry, and Optimization

Dissertation Proposal

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Mathematical optimization has been a highly active field in recent years. Typically, one seeks to leverage structure, such as convexity, when studying optimization problems. We focus on several optimization problems for which structures in the objective function or constraints are naturally phrased in the language of algebraic geometry.

The first problem we consider is regression over the space of rational functions in tropical (max-plus) algebra. Such functions form a widely expressive class of function approximators and have recently proven useful in the theoretical analysis of ReLU neural networks. We develop an alternating heuristic to solve regression problems over tropical rational functions by leveraging known results from tropical linear systems and polynomial regression. Numerical experiments show the strengths and weaknesses of the heuristic and we provide a connection between our method and geometric aspects of the loss function.

The second problem we consider is the expression of the convex hull of a set defined by three quadratic inequality constraints using nonnegative linear combinations (aggregations) of the constraints. Our approach relates the problem to the topology of the spectral curve, defined as the zero set of the determinant of linear combinations of the defining quadratics. In particular, we characterize the nonexistence of solutions to systems of inequalities in terms of the hyperbolicity of the spectral curve. Hyperbolic curves are well-studied in real algebraic geometry, as their zero sets consist of maximally nested ovals, the innermost of which bounds a convex cone. By studying (non)intersections of polyhedral cones of aggregations with hyperbolicity cones of the spectral curve, we provide a sufficient condition for the convex hull to be given by aggregations and characterize when finitely many aggregations suffice for such a description.

We will conclude by mentioning some preliminary work investigating sums of squares of real polynomials. Such polynomials are well-known to be closely connected to semidefinite programming and provide a link between convex optimization and real algebraic geometry.

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