

DISSERTATION
DEFENSE

*Reduced Unitary Whitehead Groups over Function Fields of
 p -adic Curves*

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Abstract: The study of the Whitehead group of semi-simple simply connected groups is classical with an abundance of new open questions concerning the triviality of these groups. The Kneser-Tits conjecture on the triviality of these groups was answered in the negative by Platanov for general fields. There is a relation between reduced Whitehead groups and R -equivalence classes in algebraic groups.

Let G be an algebraic group over a field F . The R -equivalence, defined by Manin, is the equivalence relation on $G(F)$ defined by $x \sim y$ for $x, y \in G(F)$ if there exists a F -rational morphism $\mathbb{A}_K^1 \cdots \rightarrow G$ defined at 0 and 1 and sending 0 to x and 1 to y . Let $RG(F)$ be the equivalence class of the identity element in $G(F)$. Then $RG(F)$ is a normal subgroup of $G(F)$ and the quotient $G(F)/RG(F)$ is called the group of R -equivalence classes of $G(F)$. It is well known that for the semi-simple simply connected isotropic group G over F , $W(G, F)$ is isomorphic to the group of R -equivalence classes. Thus the group of R -equivalence classes can be thought as Whitehead groups for general algebraic groups. The group of R -equivalence classes, is very useful while studying the rationality problem for algebraic groups, the problem to determine whether the variety of an algebraic group is rational or stably rational.

Suppose that D_0 is a central division F_0 -algebra. If the group $G(F_0)$ of rational points is given by $SL_n(D)$ for some $n > 1$, then $W(G, F_0)$ is the reduced Whitehead group of D_0 .

Let F be a quadratic field extension of F_0 and D be a central division F -algebra. Suppose that D has an involution of second kind τ such that $F^\tau = F_0$. If the hermitian form h_τ induced by τ is isotropic and the group $G(F_0)$ is given by $SU(h_\tau, D)$, then $W(G, F_0)$ is isomorphic to the reduced unitary Whitehead group of D .

We start from the fundamental facts on reduced unitary Whitehead groups of central simple algebras, then introduce the patching techniques. Finally, let F/F_0 be a quadratic field extension of the function field of a p -adic curve. Let A be a central simple algebra over F . Assume that the period of A is two and A has a unitary F/F_0 involution. We provide a proof for the triviality of the reduced unitary Whitehead group of A .

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