**Measure growth in groups and the Kemperman inverse problem**

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**Abstract:** Two perennially studied questions in arithmetic combinatorics are:  
(i) Given two sets $A, B$ of a given size, how small can $AB$ be? (ii) What structure must $A$ and $B$ have when $AB$ is as small as possible, or nearly as small as possible?  
Theorems addressing (i) are called direct theorems, and those addressing (ii) are called inverse theorems. The direct theorem for locally compact groups was obtained by Kemperman (well-known special cases include Kneser’s inequality and the Cauchy-Davenport inequality). The Kemperman inverse problem (proposed by Kemperman in 1964, also by Griesmer and Tao) corresponds to question (ii) when the ambient group is connected. In this talk, I will discuss the recent solution to this problem, highlighting the new-developed measure growth phenomenon: if $G$ is connected compact equipped with a normalized measure $\mu$, and $G$ is “sufficiently non-abelian”, $A \subseteq G$ has a sufficiently small measure, then there is a constant gap between $\mu(AA)$ and $2\mu(A)$. We then discuss a few other applications of this phenomenon, including a Brunn-Minkowski inequality in non-abelian groups. This is based on joint work with Chieu-Minh Tran and Ruixiang Zhang.

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