Analysis and PDEs Seminar

(-1)-homogeneous solutions of stationary incompressible Navier-Stokes equations with singular rays

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Abstract: In 1944, L.D. Landau first discovered explicit (-1)-homogeneous solutions of 3-d stationary incompressible Navier-Stokes equations (NSE) with precisely one singularity at the origin, which are axisymmetric with no swirl. These solutions are now called Landau solutions. In 1998 G. Tian and Z. Xin proved that all solutions which are (-1) homogeneous, axisymmetric with one singularity are Landau solutions. In 2006 V. Sverak proved that with just the (-1)-homogeneous assumption Landau solutions are the only solutions with one singularity. Our work focuses on the (-1)-homogeneous solutions of 3-d incompressible stationary NSE with finitely many singularities on the unit sphere.

In this talk we will first classify all (-1)-homogeneous axisymmetric no-swirl solutions of 3-d stationary incompressible NSE with one singularity at the south pole on the unit sphere as a two dimensional solution surface. We will then present our results on the existence of a one parameter family of (-1)-homogeneous axisymmetric solutions with non-zero swirl and smooth on the unit sphere away from the south pole, emanating from the two dimensional surface of axisymmetric no-swirl solutions. We will also present asymptotic behavior of general (-1)-homogeneous axisymmetric solutions in a cone containing the south pole with a singularity at the south pole on the unit sphere. We also constructed families of solutions smooth on the unit sphere away from the north and south poles, and will have obtained some asymptotic stability result of these solutions. This is a joint work with Professor Yanyan Li and Li Li.

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