Abstract: The group of units of a ring is one of the most basic, yet mysterious, invariants of the ring. Little is known about the structure of the unit group in general, much less explicit algorithms for computation, although the need for these do arise in applications such as tropical geometry. I will discuss some general questions about unit groups, and then specialize to the case of coordinate rings of classical algebraic varieties - in particular, describing explicit algorithms for computation in the case of smooth curves of low genus (rational and elliptic). This is based on joint work with Sameera Vemulapalli and Leon Zhang.