Abstract: Given a Galois cover of curves $X \to Y$ with Galois group $G$ which is totally ramified at a point $x$ and unramified elsewhere, restriction to the punctured formal neighborhood of $x$ induces a Galois extension of Laurent series rings $k((u))/k((t))$. If we fix a base curve $Y$, we can ask when a Galois extension of Laurent series rings comes from a global cover of $Y$ in this way. Harbater proved that over a separably closed field, every Laurent series extension comes from a global cover for any base curve if $G$ is a $p$-group, and he gave a condition for the uniqueness of such an extension. Using a generalization of Artin–Schreier theory to non-abelian $p$-groups, we characterize the curves $Y$ for which this extension property holds and for which it is unique up to isomorphism, but over a more general ground field.