# Math 346, HW5 Solution

## 3.6.1 (b)

We can introduce two artificial variables  $x_4$  and  $x_5$  to the constraint, and minimize their sum:

minimize 
$$x_4 + x_5$$
  
subject to  $x_1 + x_2 + x_4 = 1$   
 $2x_1 + x_2 - x_3 + x_5 = 3$   
 $x_1, x_2, x_3, x_4, x_5 \ge 0.$ 

Note that the objective function is equal to  $x_4 + x_5 = (1 - x_1 - x_2) + (3 - 2x_1 - x_2 + x_3) = 4 - 3x_1 - 2x_2 + x_3$ . Here are the results using simplex algorithm (in the tableau form):

Γ	$ x_1 $	$x_2$	$x_3$	$x_4$	$x_5$	7
$x_4$	1	1	0	1	0	1
$x_5$	2	1	-1	0	1	3
	-3	-2	1	0	0	-4
-						. –
Γ	$ x_1 $	$x_2$	$x_3$	$x_4$	$x_5$	1
$\left[ \begin{array}{c} \\ x_1 \end{array} \right]$	$\begin{array}{c} x_1 \\ 1 \end{array}$	$\frac{x_2}{1}$	$\frac{x_3}{0}$	$\frac{x_4}{1}$	$\frac{x_5}{0}$	1
$\begin{bmatrix} x_1 \\ x_5 \end{bmatrix}$	$\begin{array}{c c} x_1 \\ 1 \\ 0 \end{array}$	$\begin{array}{c} x_2 \\ 1 \\ -1 \end{array}$	$\begin{array}{c} x_3 \\ 0 \\ -1 \end{array}$	$\begin{array}{c} x_4 \\ 1 \\ -2 \end{array}$	$\begin{array}{c} x_5 \\ 0 \\ 1 \end{array}$	1

The simplex algorithm stops with a non-degenerate BFS such that  $x_5 > 0$ . Therefore there is no nonnegative solution satisfying the original constraints.

### 3.6.1(a)

To solve nonnegative solutions for the given constraints, we may also use the Big-M methods by choosing an arbitrary objective function, for example one can choose objective function to be  $x_1$  (0 is also fine). Now by introducing the artificial variables  $x_4, x_5$ , we would like to solve the following LP:

minimize 
$$x_1 + M(x_4 + x_5)$$
  
subject to  $x_1 - x_2 + x_4 = 1$   
 $2x_1 + x_2 - x_3 + x_5 = 3$   
 $x_1, x_2, x_3, x_4, x_5 \ge 0.$ 

In order to apply the simplex algorithm, first we rewrite the objective function as a function only in the non-basic variables  $x_1, x_2, x_3$ :

$$x_1 + M(x_4 + x_5) = x_1 + M(1 - x_1 + x_2) + M(3 - 2x_1 - x_2 + x_3)$$
  
= 4M + (1 - 3M)x\_1 + Mx\_3

Γ	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	-
$x_4$	1	-1	0	1	0	1
$x_5$	2	1	-1	0	1	3
	1 - 3M	0	M	0	0	-4M

The 1 - 3M term in the last row (objective function) is negative, so we can do minimum ratio test for the first column, and pivot the entry 1:

Γ		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
x	1	1	-1	0	1	0	1
x	5	0	3	-1	-2	1	1
		0	1 - 3M	M	3M - 1	0	-M - 1

The 1 - 3M term is still negative, we apply simplex method one more round:

ſ	•	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	-
	$x_1$	1	0	-1/3	1/3	1/3	4/3
	$x_2$	0	1	-1/3	-2/3	1/3	1/3
	_	0	0	1/3	M - 1/3	M - 1/3	-4/3

Now all the artificial variables are out of the basis, this gives us a feasible solution to the original LP, with  $(x_1, x_2, x_3) = (4/3, 1/3, 0)$ .

## 3.6.2(c)

We use the two-phase method. In the first phase, we minimize the sum of artificial variables:

minimize 
$$x_5 + x_6$$
  
subject to  $4x_1 + x_2 + x_3 + 4x_4 + x_5 = 8$   
 $x_1 - 3x_2 + x_3 + 2x_4 + x_6 = 16$   
 $x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$ 

We can choose  $x_5$  and  $x_6$  as the initial basis. The objective function is thus equal to  $x_5 + x_6 = (8 - 4x_1 - x_2 - x_3 - 4x_4) + (16 - x_1 + 3x_2 - x_3 - 2x_4) = 24 - 5x_1 + 2x_2 - 2x_3 - 6x_4.$ 

Г		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	-
x	5	4	1	1	4	1	0	8
x	6	1	-3	1	2	0	1	16
		-5	2	-2	-6	0	0	-24

Γ	-	$x_1$	x	2	$x_3$	$x_4$	$x_{5}$	5	$x_6$		-
	$x_4$	1	1/	/4	1/4	1	1/	4	0	، 4	2
	$x_6$	-1	-7	$^{\prime}/2$	1/2	0	$-1_{1}$	/2	1	1	2
		1	7/	2	-1/2	0	3/	2	0	—	12
		-	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$		٦	
		$x_3$	4	1	1	4	1	0	8	3	
		$x_6$	-3	-4	0	-2	-1	1	8	3	
			3	4	0	2	2	0	-	8	

Note that this is a non-degenerate optimal solution with the artificial variable  $x_6 > 0$ , therefore the original LP is infeasible (so we do not have to continue the second phase).

# 3.7.2

(a) Note that the sum of the first two constraints give  $x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} = 900$ , while the sum of the last three constraints give the same expression. This gives a relationship between the five equations, which shows that one of the equations is redundant.

(b) We would like to solve the following linear program (phase I):

$$\begin{array}{lll} \text{minimize} & a_1+a_2+a_3+a_4+a_5\\ \text{subject to} & x_{11}+x_{12}+x_{13}+a_1=350\\ & x_{21}+x_{22}+x_{23}+a_2=550\\ & x_{11}+x_{21}+a_3=275\\ & x_{12}+x_{22}+a_4=325\\ & x_{13}+x_{23}+a_5=300\\ & x_{ij},a_i\geq 0. \end{array}$$

We start with  $a_i$ 's as the basic variable and write the objective function in the non-basic variable, therefore we could start with the following tableau:

	「	$x_{11}$	$x_{12}$	$x_{13}$	$x_{21}$	$x_{22}$	$x_{23}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	]
	$a_1$	1	1	1	0	0	0	1	0	0	0	0	350
	$a_2$	0	0	0	1	1	1	0	1	0	0	0	550
	$a_3$	1	0	0	1	0	0	0	0	1	0	0	275
	$a_4$	0	1	0	0	1	0	0	0	0	1	0	325
	$a_5$	0	0	1	0	0	1	0	0	0	0	1	300
		-2	-2	-2	-2	-2	-2	0	0	0	0	0	-1800
г		I											
		$x_{11}$	rea	m									
		~11	$x_{12}$	$x_{13}$	$x_{21}$	$x_{22}$	$x_{23}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	
	$a_1$	0	$\frac{x_{12}}{1}$	$\frac{x_{13}}{1}$	$\frac{x_{21}}{-1}$	$\frac{x_{22}}{0}$	$\frac{x_{23}}{0}$	$\frac{a_1}{1}$	$\frac{a_2}{0}$	$\frac{a_3}{-1}$	$\frac{a_4}{0}$	$\frac{a_5}{0}$	75
	$a_1 \\ a_2$		$\frac{x_{12}}{1}$	$\begin{array}{c} x_{13} \\ 1 \\ 0 \end{array}$						-	-	-	75 550
	-	0	1	1	-1	0	0	1	0	-1	0	0	
	$a_2$	0 0	1 0	1 0	$-1 \\ 1$	$\begin{array}{c} 0 \\ 1 \end{array}$	0 1	$\begin{array}{c} 1 \\ 0 \end{array}$	0 1	$-1 \\ 0$	0 0	0 0	550
	$a_2$ $x_{11}$	0 0 1	1 0 0	1 0 0	-1 1 1	0 1 0	0 1 0	$\begin{array}{c} 1 \\ 0 \\ 0 \end{array}$	0 1 0	$-1 \\ 0 \\ 1$	0 0 0	0 0 0	$550 \\ 275$

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		<i>x</i> <sub>11</sub>	<i>x</i> <sub>12</sub>	$x_{13}$	$x_{21}$	<i>x</i> <sub>22</sub>	<i>x</i> <sub>23</sub>	<i>a</i> <sub>1</sub>	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>	<i>a</i> <sub>4</sub>	$a_5$	
$x_{12}$	2	0	1	1	-1	0	0	1	0	-1	0	0	75
$a_2$	2	0	0	0	1	1	1	0	1	0	0	0	550
$x_1$	1	1	0	0	1	0	0	0	0	1	0	0	275
$a_4$	L	0	0	-1	1	1	0	-1	0	1	1	0	250
$a_5$	5	0	0	1	0	0	1	0	0	0	0	1	300
L		0	0	0	-2	-2	-2	2	0	0	0	0	-1100
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		$x_{11}$	<i>x</i> <sub>12</sub>	<i>x</i> <sub>13</sub>	x <sub>21</sub>	x <sub>22</sub>	x <sub>23</sub>	$a_1$	$a_2$	<i>a</i> <sub>3</sub>	<i>a</i> <sub>4</sub>	<i>a</i> <sub>5</sub>	225
$x_1$	2	0	1	0	0	1	0	0	0	0	1	0	325
$a_2$	2	0	0	1	0	0	1	1	1	-1	-1	0	300
$x_1$	1	1	0	1	0	-1	0	1	0	0	-1	0	25
$x_2$	1	0	0	-1	1	1	0	-1	0	1	1	0	250
$a_5$	5	0	0	1	0	0	1	0	0	0	0	1	300
		0	0	-2	0	0	-2	0	0	0	2	2	-600
- г		~	~	~	~	~	~	~	~	~	~	~	I 7
- [		$\frac{x_{11}}{2}$	x <sub>12</sub>	$x_{13}$	$\frac{x_{21}}{2}$	x <sub>22</sub>	x <sub>23</sub>	$a_1$	$a_2$	$a_3$	<i>a</i> <sub>4</sub>	$a_5$	205
$\boxed{x_1}$		0	1	0	0	1	0	0	0	0	1	0	325
$a_2$	2	$0 \\ -1$	$\begin{array}{c} 1 \\ 0 \end{array}$	0 0	0 0	1 1	$\begin{array}{c} 0 \\ 1 \end{array}$	0 0	0 1	$0 \\ -1$	1 0	0 0	275
	2		1 0 0	0 0 1	0 0 0	1 1 -1	0 1 0	0 0 1	0 1 0	$0 \\ -1 \\ 0$	$     \begin{array}{c}       1 \\       0 \\       -1     \end{array} $	0 0 0	$275 \\ 25$
$a_2$	2 .3	$     \begin{array}{c}       0 \\       -1 \\       1 \\       1     \end{array} $	1 0 0 0	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array}$	$     \begin{array}{c}       0 \\       0 \\       0 \\       1     \end{array} $	$     \begin{array}{c}       1 \\       1 \\       -1 \\       0     \end{array} $	0 1 0 0	0 0 1 0	0 1 0 0	0 -1 0 1	$     \begin{array}{c}       1 \\       0 \\       -1 \\       0     \end{array} $	0 0 0 0	$275 \\ 25 \\ 275$
$\begin{vmatrix} a_2 \\ x_1 \end{vmatrix}$	2 .3 :1	$0 \\ -1 \\ 1 \\ 1 \\ -1$	$     \begin{array}{c}       1 \\       0 \\       0 \\       0 \\       0     \end{array} $	0 0 1 0 0	0 0 0 1 0	$     \begin{array}{c}       1 \\       1 \\       -1 \\       0 \\       1     \end{array} $	0 1 0 0 1	$0 \\ 0 \\ 1 \\ 0 \\ -1$	0 1 0 0 0	$\begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{array}$	$     \begin{array}{c}       1 \\       0 \\       -1 \\       0 \\       1     \end{array} $	0 0 0 0 1	$275 \\ 25 \\ 275 \\ 275 \\ 275$
$\begin{vmatrix} a_2 \\ x_1 \\ x_2 \end{vmatrix}$	2 .3 :1	$     \begin{array}{c}       0 \\       -1 \\       1 \\       1     \end{array} $	1 0 0 0	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array}$	$     \begin{array}{c}       0 \\       0 \\       0 \\       1     \end{array} $	$     \begin{array}{c}       1 \\       1 \\       -1 \\       0     \end{array} $	0 1 0 0	0 0 1 0	0 1 0 0	0 -1 0 1	$     \begin{array}{c}       1 \\       0 \\       -1 \\       0     \end{array} $	0 0 0 0	$275 \\ 25 \\ 275$
$\begin{vmatrix} a_2 \\ x_1 \\ x_2 \end{vmatrix}$	2 .3 :1	$0 \\ -1 \\ 1 \\ 1 \\ -1 \\ 2$	$     \begin{array}{c}       1 \\       0 \\       0 \\       0 \\       0 \\       0     \end{array} $	0 0 1 0 0	$egin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array}$	$     \begin{array}{c}       1 \\       1 \\       -1 \\       0 \\       1 \\       -2     \end{array} $	$     \begin{array}{c}       0 \\       1 \\       0 \\       0 \\       1 \\       -2     \end{array} $		0 1 0 0 0 0	$     \begin{array}{c}       0 \\       -1 \\       0 \\       1 \\       0 \\       2     \end{array} $	$     \begin{array}{c}       1 \\       0 \\       -1 \\       0 \\       1 \\       0     \end{array} $	0 0 0 0 1 0	275 25 275 275 -550
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 1 5	$ \begin{array}{c}     0 \\     -1 \\     1 \\     -1 \\     2 \\     x_{11} \end{array} $	$egin{array}{cccc} 1 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ x_{12} \end{array}$	$egin{array}{ccc} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ x_{13} \end{array}$	$egin{array}{ccc} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{21} \end{array}$	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ \end{array}$	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_2 \end{array} $	$ \begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 2 \\ a_3 \end{array} $	$\begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \end{array}$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ a_5 \end{array}$	275 25 275 275 -550
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 5 1 5	$ \begin{array}{c} 0 \\ -1 \\ 1 \\ -1 \\ 2 \\ x_{11} \\ 1 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ x_{12} \\ 1 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{13} \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ x_{21} \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \\ -1 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ \end{array}$ $\begin{array}{c} 2 \\ a_1 \\ 0 \end{array}$	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ a_2 \\ -1 \end{array} $	$\begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ \end{array}$ $\begin{array}{c} 2 \\ a_3 \\ 1 \end{array}$	$\begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \\ 1 \end{array}$		$ \begin{array}{c c} 275 \\ 25 \\ 275 \\ 275 \\ -550 \\ \hline 50 \\ \end{array} $
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 11 5 12 22	$ \begin{array}{c} 0 \\ -1 \\ 1 \\ -1 \\ 2 \\ x_{11} \\ 1 \\ -1 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ x_{12} \\ 1 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{13} \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ x_{21} \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \\ 0 \\ 1 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \\ -1 \\ 1 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 2 \\ a_1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_2 \\ -1 \\ 1 \end{array} $	$\begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 2 \\ a_3 \\ 1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \\ 1 \\ 0 \end{array}$		$ \begin{array}{c} 275 \\ 25 \\ 275 \\ 275 \\ -550 \\ \hline 50 \\ 275 \\ \hline 275 \\ \hline \end{array} $
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 5 12 12 22 13	$ \begin{array}{c} 0 \\ -1 \\ 1 \\ -1 \\ 2 \\ x_{11} \\ -1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ x_{12} \\ 1 \\ 0 \\ 0 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{13} \\ 0 \\ 0 \\ 1 \\ \end{array}$	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{21} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \\ 0 \\ 1 \\ 0 \\ \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \\ -1 \\ 1 \\ 1 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 2 \\ a_1 \\ 0 \\ 0 \\ 1 \\ \end{array} $	$egin{array}{ccc} 0 & & & \ 1 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ a_2 & & \ 1 & & \ 1 & & \ 1 & \ 1 & \ \end{array}$	$ \begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 2 \\ a_3 \\ 1 \\ -1 \\ -1 \end{array} $	$\begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \\ 1 \\ 0 \\ -1 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ a_5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$ \begin{array}{c} 275 \\ 25 \\ 275 \\ 275 \\ -550 \\ \hline 50 \\ 275 \\ 300 \\ \end{array} $
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 11 5 12 22 13 21	$ \begin{array}{c} 0 \\ -1 \\ 1 \\ 1 \\ -1 \\ 2 \\ x_{11} \\ 1 \\ -1 \\ 0 \\ 1 \\ \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ x_{12} \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \end{array} \\ \begin{array}{c} x_{13} \\ 0 \\ 0 \\ 1 \\ 0 \\ \end{array}$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \end{array} \\ x_{21} \\ 0 \\ 0 \\ 0 \\ 1 \\ \end{array}$	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \\ 0 \\ 1 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \\ -1 \\ 1 \\ 0 \\ \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 2 \\ a_1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} $	$egin{array}{cccc} 0 & & & \ 1 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ 1 & & \ 1 & & \ 1 & & \ 0 & & \$	$\begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 2 \\ a_3 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{array}$	$ \begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{array} $	$egin{array}{ccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $	$ \begin{array}{c} 275 \\ 25 \\ 275 \\ 275 \\ -550 \\ \hline 50 \\ 275 \\ 300 \\ 275 \\ 300 \\ 275 \\ \end{array} $
$\begin{bmatrix} a_2 \\ x_1 \\ x_2 \\ a_5 \end{bmatrix}$	2 3 11 5 12 22 13 21	$ \begin{array}{c} 0 \\ -1 \\ 1 \\ -1 \\ 2 \\ x_{11} \\ -1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ x_{12} \\ 1 \\ 0 \\ 0 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{13} \\ 0 \\ 0 \\ 1 \\ \end{array}$	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ x_{21} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 0 \\ 1 \\ -2 \\ x_{22} \\ 0 \\ 1 \\ 0 \\ \end{array} $	$ \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ -2 \\ x_{23} \\ -1 \\ 1 \\ 1 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 2 \\ a_1 \\ 0 \\ 0 \\ 1 \\ \end{array} $	$egin{array}{ccc} 0 & & & \ 1 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ 0 & & \ a_2 & & \ 1 & & \ 1 & & \ 1 & \ 1 & \ \end{array}$	$ \begin{array}{c} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 2 \\ a_3 \\ 1 \\ -1 \\ -1 \end{array} $	$\begin{array}{c} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ a_4 \\ 1 \\ 0 \\ -1 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ a_5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$ \begin{array}{c} 275 \\ 25 \\ 275 \\ 275 \\ -550 \\ \hline 50 \\ 275 \\ 300 \\ \end{array} $

The first phase of the simplex method stops here. Note that the artificial variable  $a_5$  is still in the basis, and we cannot take it out from the basis by replacing it by a non-artificial variable. Therefore we can conclude that the constraints are redundant (in particular, the fifth constraint can be written as a linear combination of the other constraints).