# Math 346, HW1 Solution

2.2.8

Suppose we use x lbs of the premium loam and y lbs of the generic loam for every 50 lbs. The cost per 50 lbs is $5x + y$, the weight of soil it contains is $0.6x + 0.2y$, the domestic manure it contains is equal to $0.4x + 0.1y$. Using the constraints in the problem, we can formulate the following linear program:

$$
\begin{array}{ll}
\text{minimize} & 5x + y \\
\text{subject to} & 0.6x + 0.2y \geq 0.36 \times 50 = 18 \\
& 0.4x + 0.1y \geq 0.2 \times 50 = 10 \\
& x + y = 50 \\
& x \geq 0, y \geq 0.
\end{array}
$$
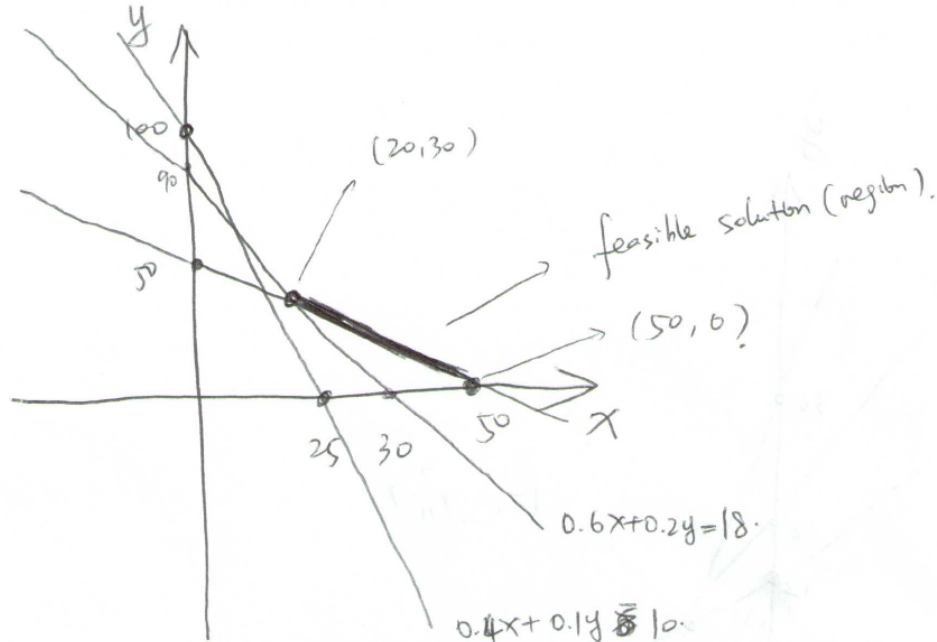
We draw the pictures as above and the feasible region is the line segment connecting $(20, 30)$ and $(50, 0)$. Here $x = 20$, $y = 30$ is the solution of $0.6x + 0.2y =$

18, $x+y = 50$, i.e. the intersecting point of these two lines. Comparing the value of objective function at this two points: $5 \times 20 + 30 = 130$ versus $5 \times 50 + 0 = 250$. Therefore the optimal solution is $x = 20$, $y = 30$, so the minimum cost per 50 lbs is $5x + y = 130$. And the optimal way is to use 20 lbs of the premium loam and 30 lbs of the generic loam (or equivalently, 40% of permium loam plus 60% of generic loam).

2.2.12

Suppose in 100 lb of lawn fertilizer, we use A lb of chemical A, $\cdots$, E lb of chemical E. Then by computing the requirement for percentages of nitrogen, phosphoric acid, potash we get the following linear program:

$$
\begin{aligned}
\text{minimize} \quad & 0.1A + 0.23B + 0.1C + 0.3D + 0.15E \\
\text{subject to} \quad & 0.18A + 0.28B + 0.3D + 0.16E = 23 \\
& 0.12A + 0.05B + 0.06C + 0.07D + 0.03E = 7 \\
& 0.05B + 0.18C + 0.08D + 0.02E = 7 \\
& A + B + C + D + E = 100 \\
& A, B, C, D, E \geq 0.
\end{aligned}
$$

This linear program above is for the case you want the percentage of nitrogen, phosphoric acid and potash to be exactly those numbers. If you only need the percentage to exceed those numbers, the equalities should be changed to $\geq$.

2.3.7

Suppose the plant spend x hours on Process 1 and y hours on Process 2. Then by considering the production requirements for the two products A and B, we can establish the following linear program:

$$
\begin{aligned}
\text{minimize} \quad & 25x + 20y \\
\text{subject to} \quad & 3x + 5y \geq 90 \\
& 6x + 5y \geq 120 \\
& x, y \geq 0.
\end{aligned}
$$
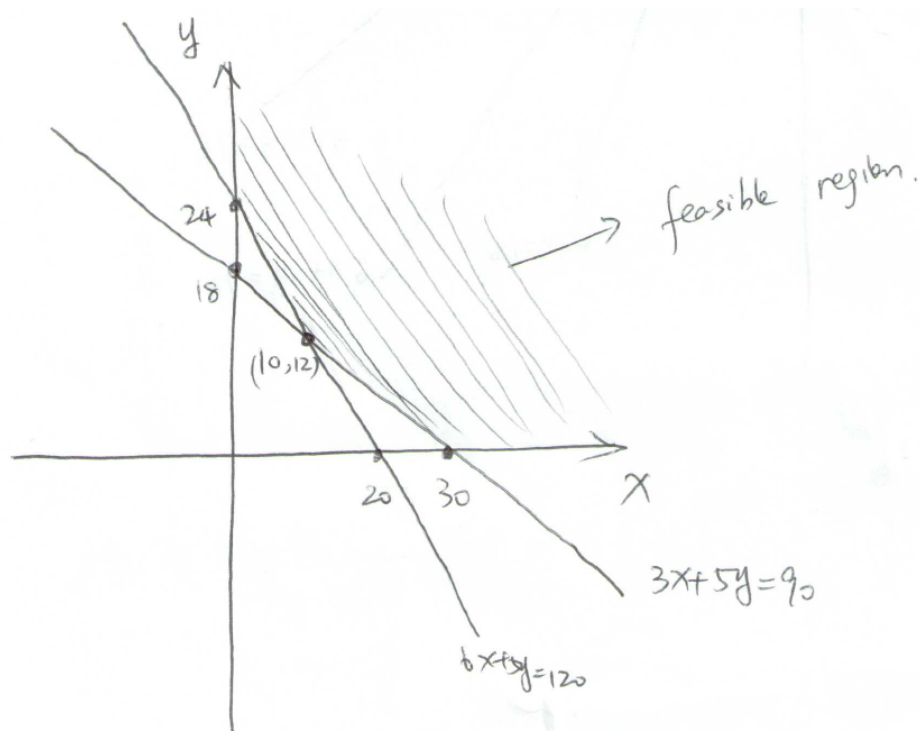
The feasible region is the area in the first quadrant that lies above the two lines. They intersect at the point $(10, 12)$. Therefore we only need to compare the values of the objective function at the points $(0, 24)$, $(10, 12)$ and $(30, 0)$. By calculation, the optimal solution is $x = 0$, $y = 24$, meaning that we need to spend all the available hours on the second process. The optimal cost is equal to $25 \times 0 + 24 \times 20 = 480$.

2.3.16

We assume that the company uses Machine i for x_i hours. Then the raw material needed is $80x_1 + 50x_2 + 76x_3$ lb. And the labor needed is $16x_1 + 35x_2 + 33x_3$ worker-hours. Suppose the company would pay for M_1 (resp. M_2) lb of raw materials over (resp. under) 1 ton (=2000 lb), and L_1 (resp. L_2) hours of labor under (resp. over) 900 hours.

$$
\begin{aligned}
\text{minimize} \quad & 4M_1 + 5.5M_2 + 8L_1 + 12L_2 \\
\text{subject to} \quad & 80x_1 + 50x_2 + 76x_3 \leq M_1 + M_2 \\
& 16x_1 + 35x_2 + 33x_3 \leq L_1 + L_2 \\
& 37x_1 + 43x_2 + 52x_3 \geq 2000 \\
& M_1 \leq 2000, L_1 \leq 900, L_2 \leq 200 \\
& x_1, x_2, x_3, M_1, M_2, L_1, L_2 \geq 0.
\end{aligned}
$$

This formulation works because the cost for raw materials exceeding 2000 lb is more expensive, also the overtime costs more than regular labor. So in the optimal solution to this minimization problem, we always max out M_1 and L_1 before M_2 and L_2 becomes strictly positive. It is also correct if we change the inequalities in the first two constraints into equalities.

2.3.23

(a) Assume that the shop produces x chairs and y sofas, then the linear program is as below (note that we have to maximize the profit, so it is a maximization problem)

$$
\begin{aligned}
\text{maximize} \quad & 70x + 60y \\
\text{subject to} \quad & 3x + 8y \le 96 \\
& 6x + 5y \le 90 \\
& 9x + 4y \le 120 \\
& x, y \ge 0.
\end{aligned}
$$

(b) Solving the LP using graphical method:
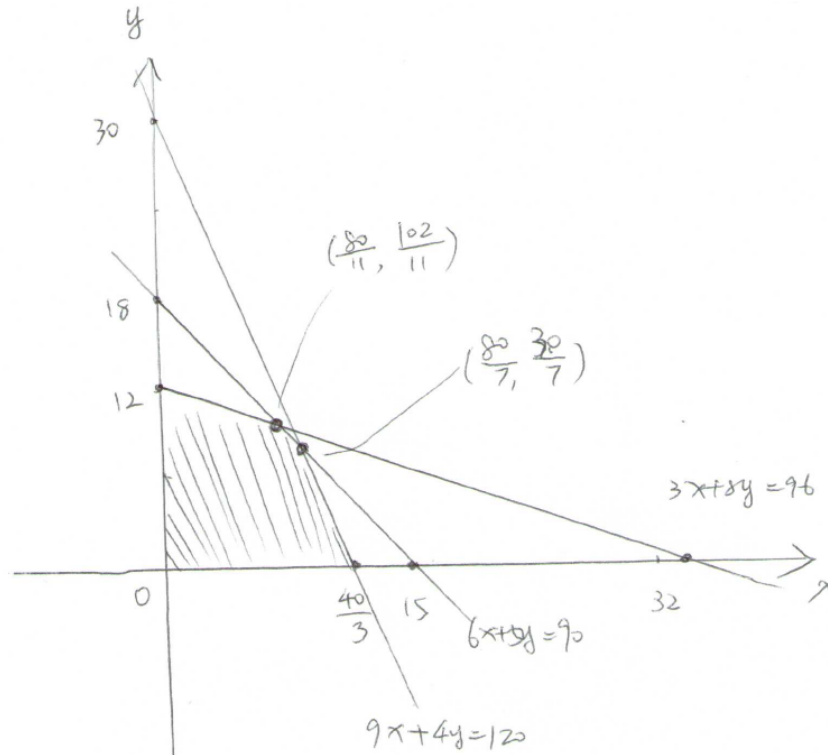
From the picture we know the maximum is achieved among one of the four points: $(0, 12)$, $(80/11, 102/11)$, $(80/7, 30/7)$ and $(40/3, 0)$. Comparing the values of the objective function at these points, we get the optimal solution $(x, y) = (7\frac{3}{11}, 9\frac{3}{11})$.

(c) (d) To implement such a schedule, we might round up or down the non-integral optimal solutions. For example we can take $(x, y) = (7, 9)$ or $(7, 10)$ or $(8, 9)$ or $(8, 10)$. For $(x, y) = (8, 10)$, $3x + 8y = 24 + 80 = 104 > 96$ thus it is not feasible. For $(x, y) = (8, 9)$, $6x + 5y = 48 + 45 = 93 > 90$ which is also not feasible. Finally for $(x, y) = (7, 10)$, we have $6x + 5y = 42 + 50 = 92 > 90$, again not feasible. The only possible choice is $(x, y) = (7, 9)$, which satisfies all the constraints and gives $70x + 60y = 490 + 540 = 1030$.

(e) However, if we let $(x, y) = (10, 6)$, again we can check this is a feasible solution to the linear program, while it gives $70x + 60y = 700 + 360 = 1060$, which is better than the solution we obtain by rounding up or down the non-integral optimal solution. This example shows that for an integer program, it is usually not sufficient to solve the corresponding linear program (by relaxing all the integral variables to be any real number).